A Graph Cut Approach to Image Segmentation in Tensor Space

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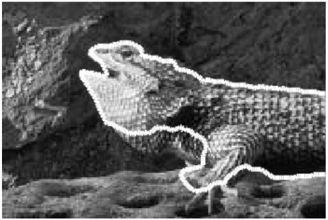

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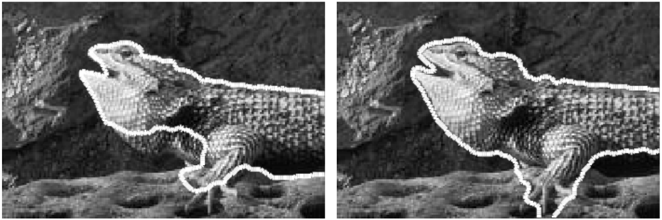

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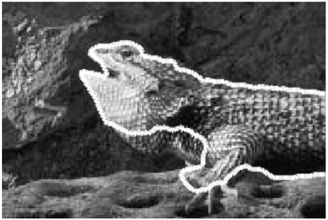

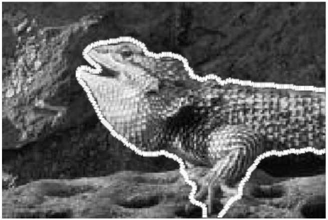

- Tensor feature space and structure

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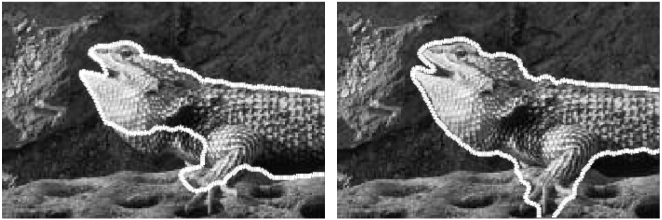

- Tensor feature space and structure
- Graph cut segmentation

Outline

So what are tensors?

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- Symmetric positive-definite matrices

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- Capture structural information

Examples:

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- Structure tensor from smoothed image derivatives

$$\mathbf{T} = K_\rho * [\mathcal{I}_x \ \mathcal{I}_y]^T [\mathcal{I}_x \ \mathcal{I}_y] = \begin{pmatrix} K_\rho * \mathcal{I}_x^2 & K_\rho * \mathcal{I}_x \mathcal{I}_y \\ K_\rho * \mathcal{I}_x \mathcal{I}_y & K_\rho * \mathcal{I}_y^2 \end{pmatrix}$$

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- Structure tensor across color channels

$$\mathbf{T} = \sum_{i=1}^{N} \left(K_\rho * [\mathcal{I}_x^i \; \mathcal{I}_y^i]^T [\mathcal{I}_x^i \; \mathcal{I}_y^i] \right)$$

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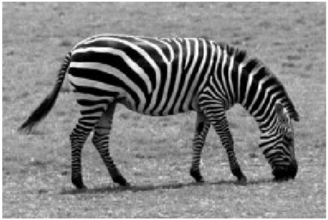
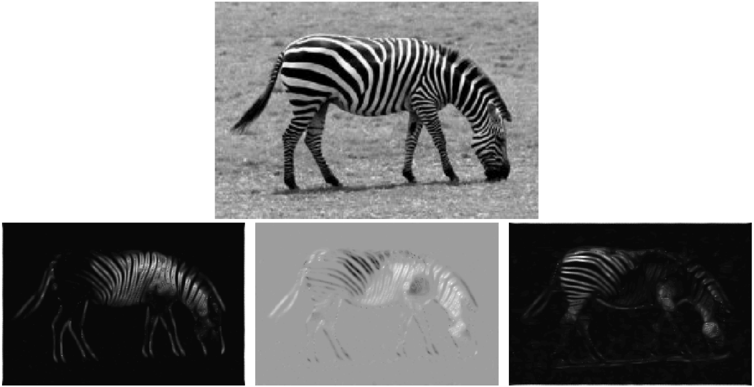
- Incorporating intensity

$$\mathbf{T} = K_\rho * [\mathcal{I} \ \mathcal{I}_x \ \mathcal{I}_y]^T [\mathcal{I} \ \mathcal{I}_x \ \mathcal{I}_y].$$

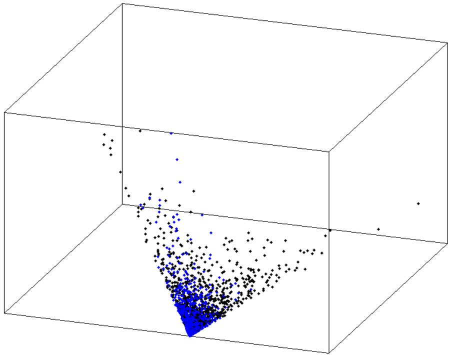


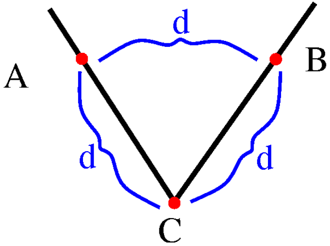


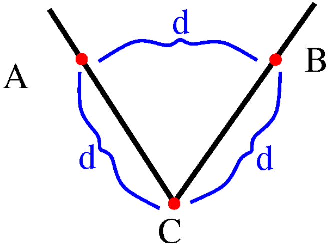
$$\mathcal{I}_x^2 \quad \mathcal{I}_x\mathcal{I}_y \quad \mathcal{I}_y^2$$

Outline

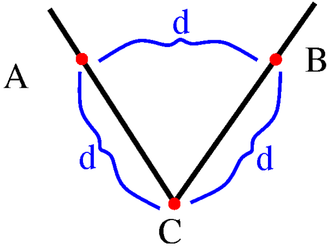
Conical surface structure



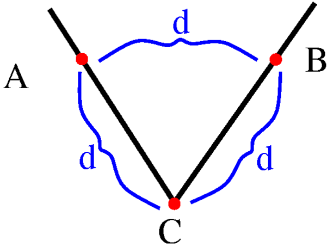

d
A
B
d
d
C



$$d(A, B) =$$



$$d(A, B) = d(A, C) + d(C, B) = 2d$$



$$d(A,B) = d(A,C) + d(C,B) = 2d \neq d$$

Why do we care?

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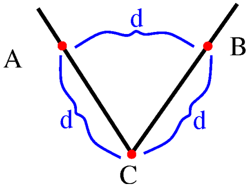

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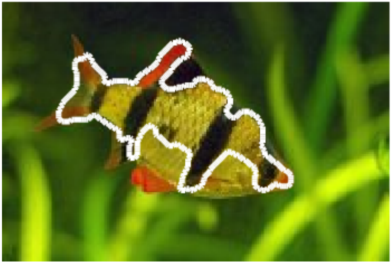

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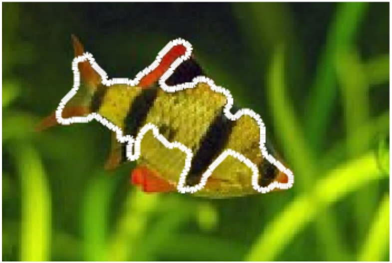 

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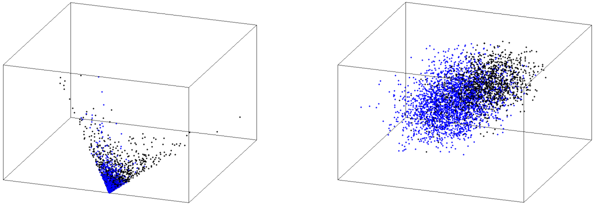

Outline

Standard binary graph cut energy formulation:

$$E(A) = \sum_{p \in \mathcal{I}} R_p(A_p) + \lambda \sum_{(p,q) \in \mathcal{N}} B_{(p,q)}$$

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Tensor version:

$$E(A) = \sum_{p \in \mathcal{I}} R_p(A_p) + \lambda \sum_{(p,q) \in \mathcal{N}} B_{(p,q)}$$

$$R_p(\mathcal{F}) = -\ln P(\mathbf{T}_p | \mathcal{F}) \qquad R_p(\mathcal{B}) = -\ln P(\mathbf{T}_p | \mathcal{B})$$

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4. Graph cut segmentation

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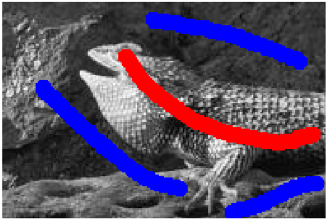

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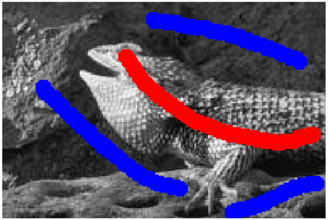

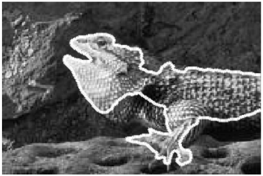

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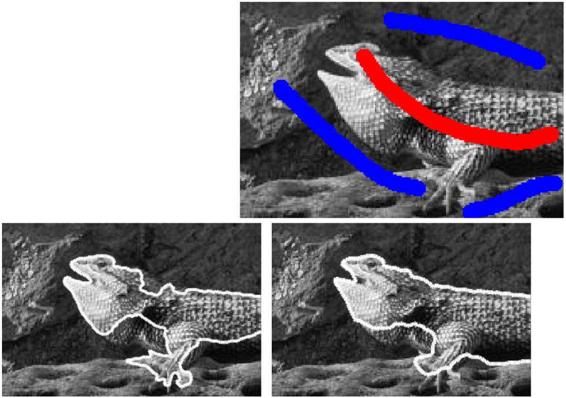

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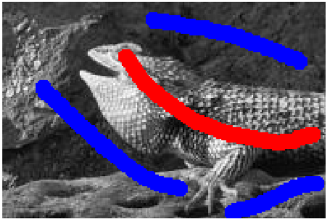

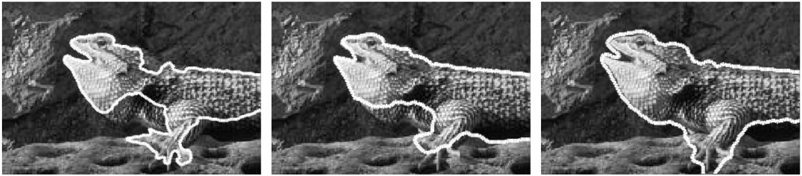

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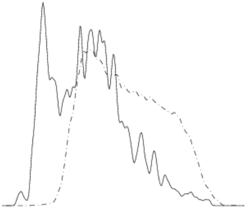

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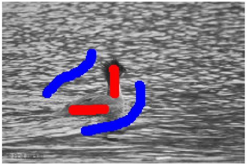

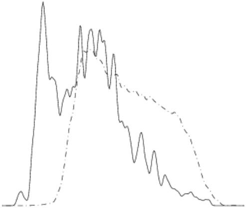

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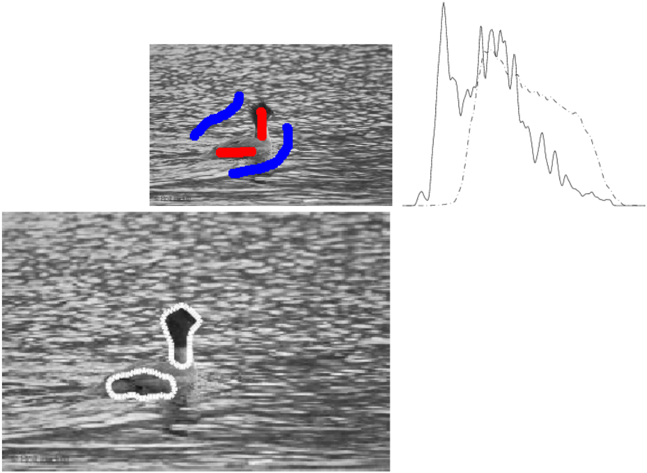

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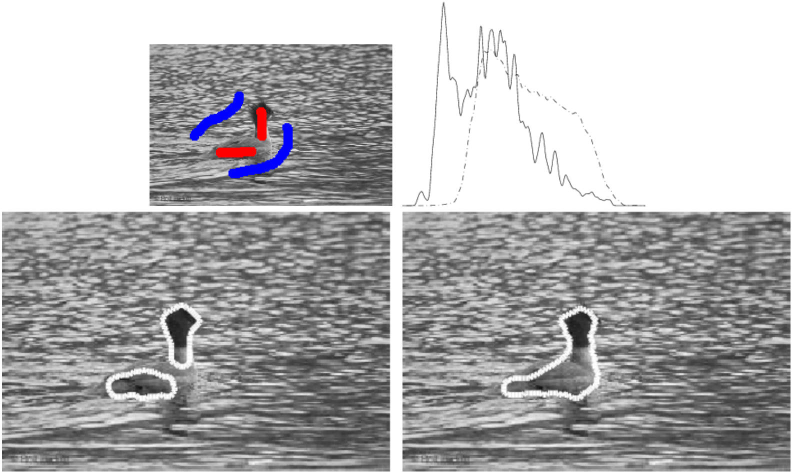

Ignoring conical structure

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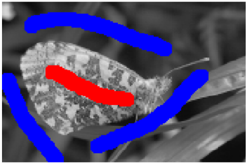

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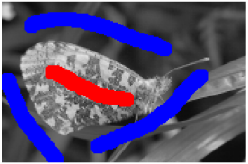

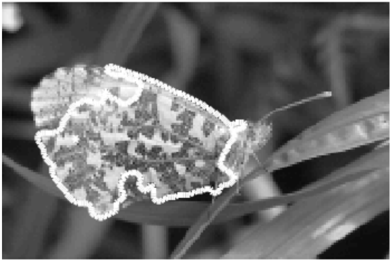

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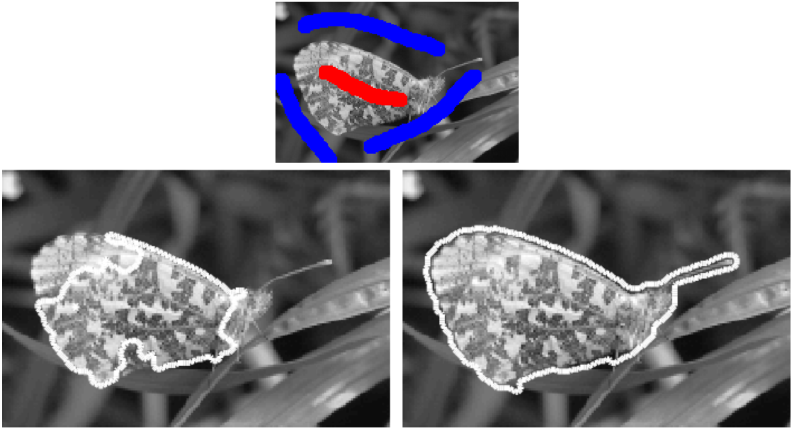

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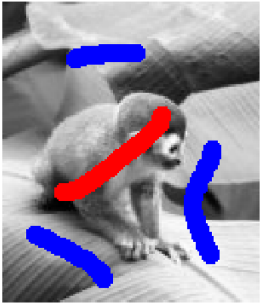

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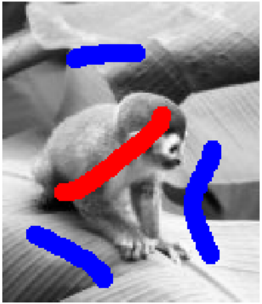

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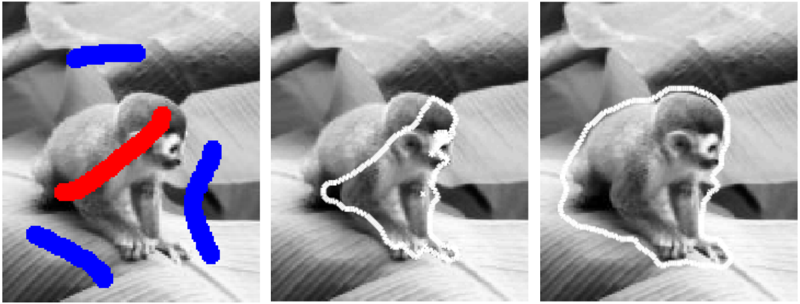
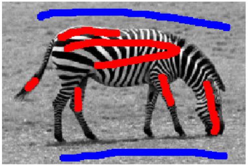


Validation, incorporate intensity



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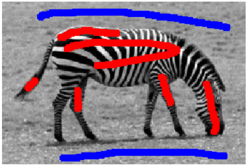

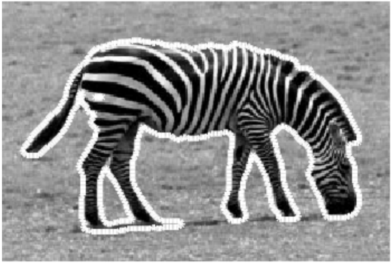

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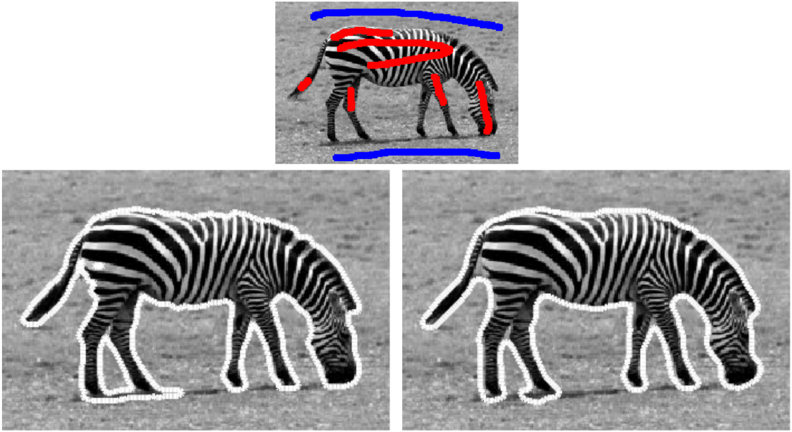
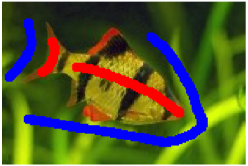


Using color



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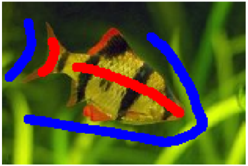

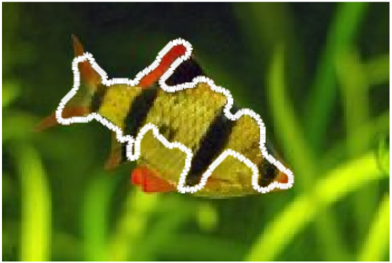

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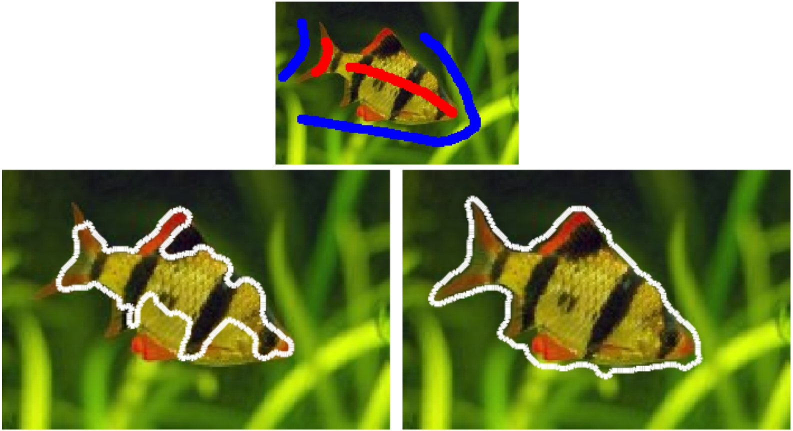

Map to identity (Log-Euclidean space)

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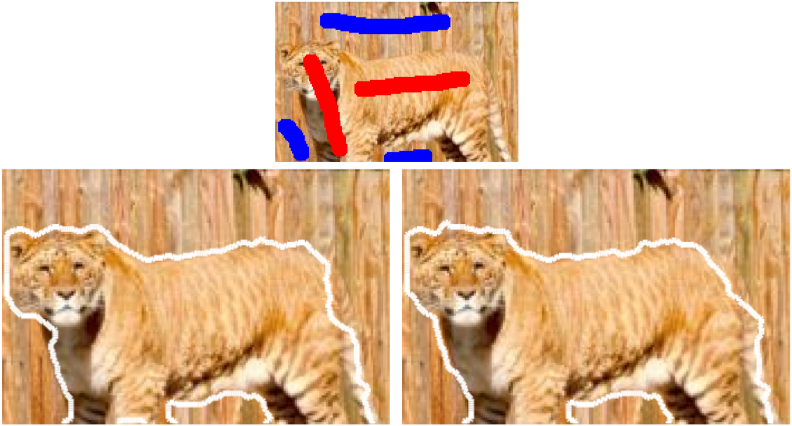
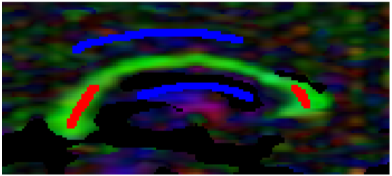
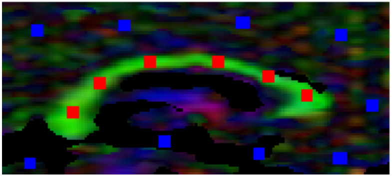


Diffusion tensor MRI





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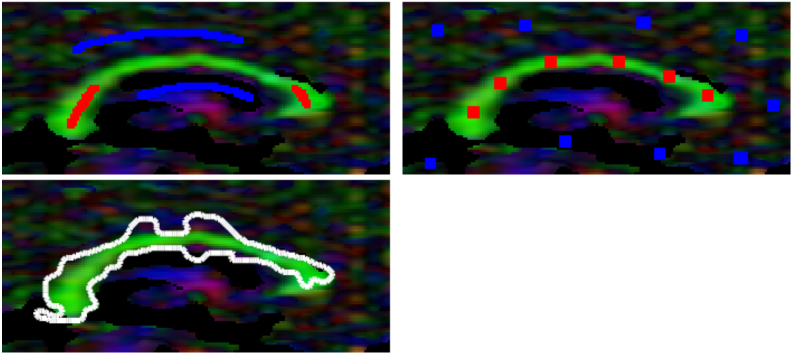

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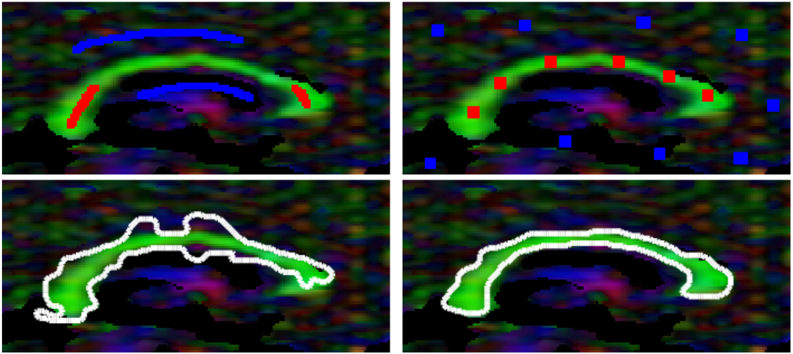

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Questions?