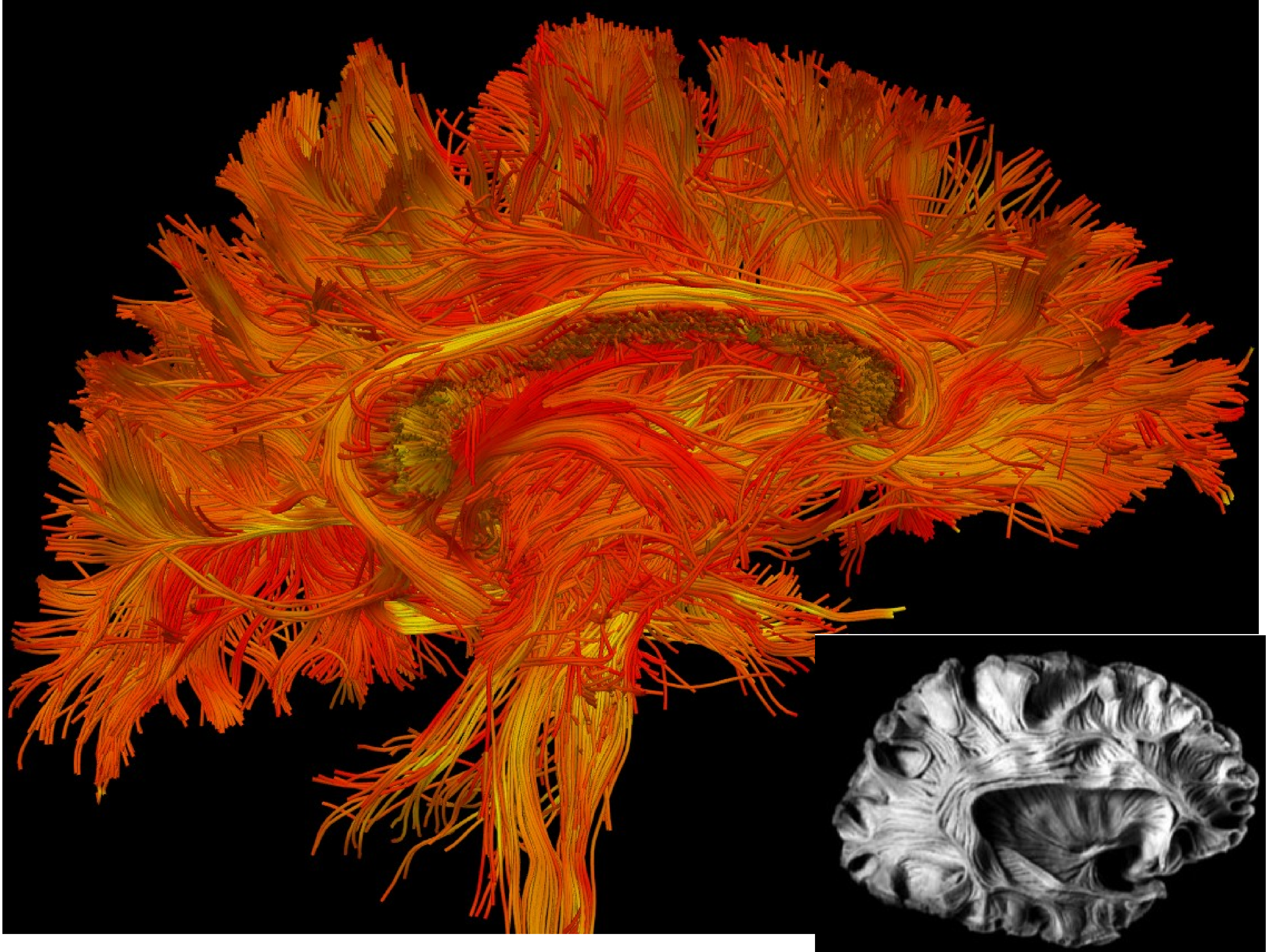
Neural Tractography Using an Unscented Kalman Filter

James Malcolm

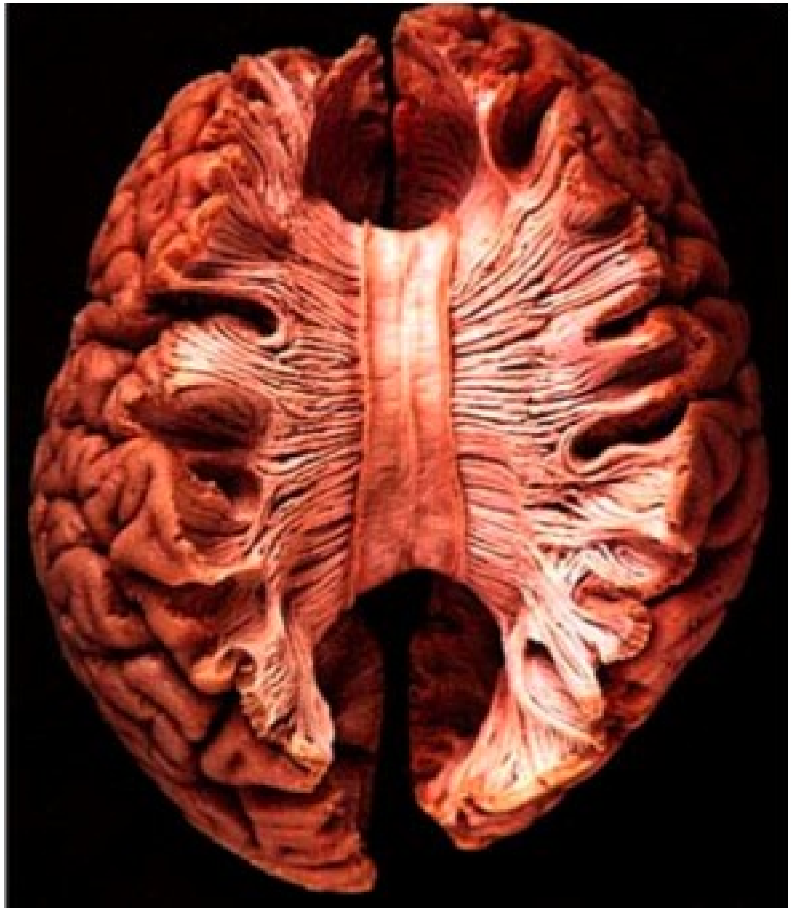
PhD Defense
21 Oct 2010

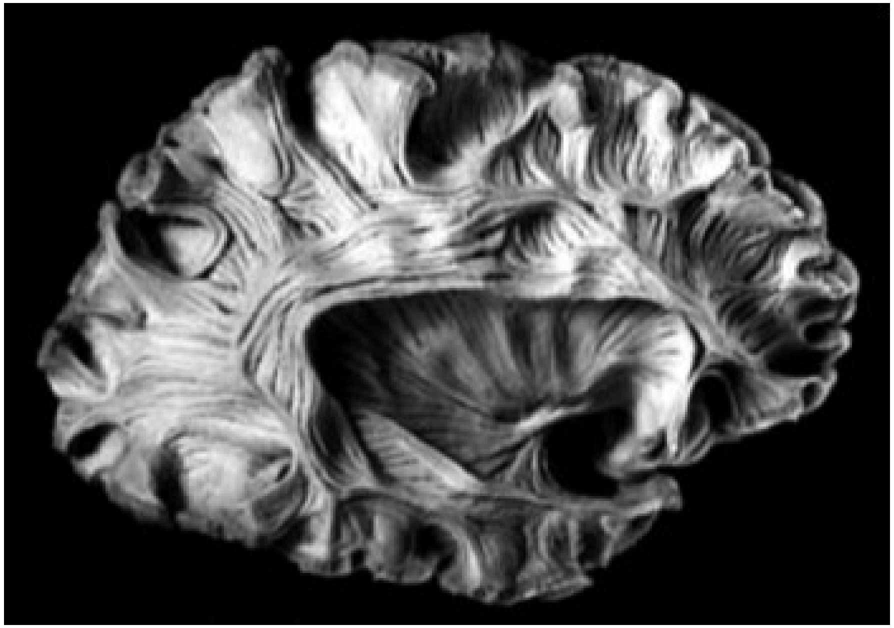


background

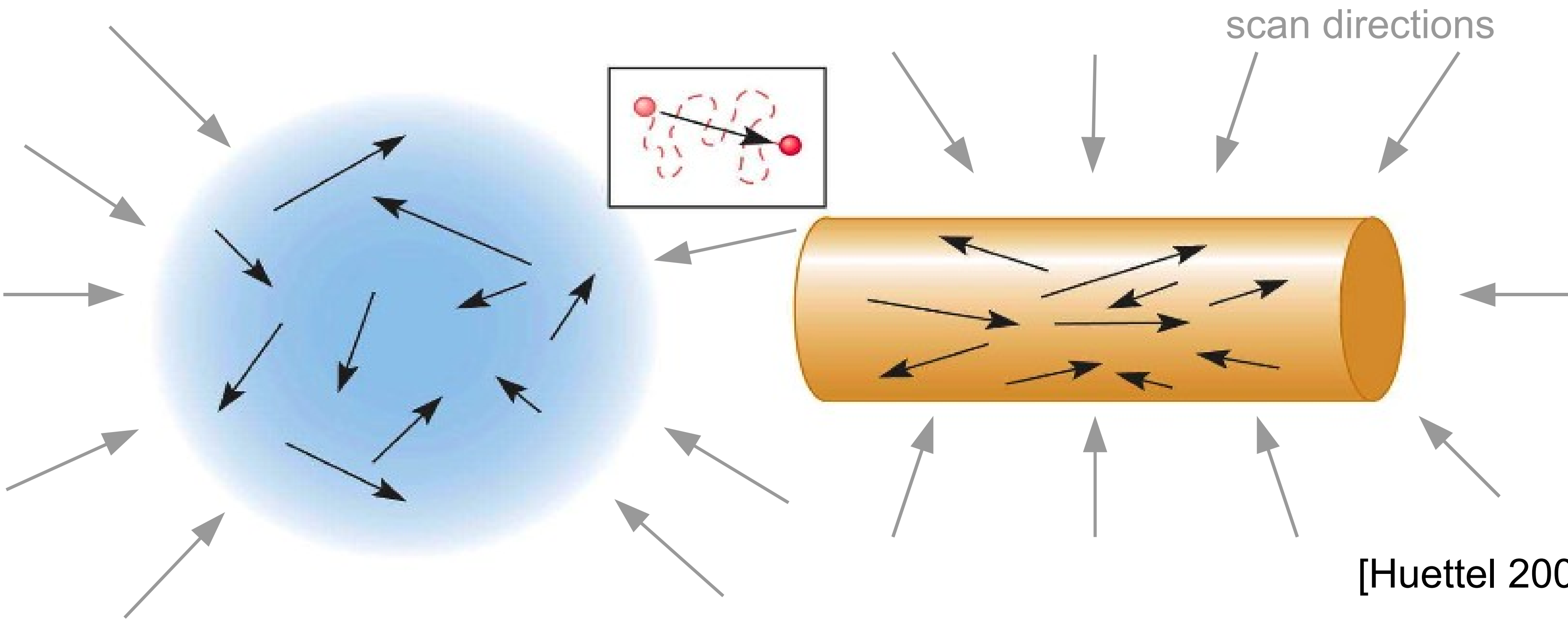
J. G. Malcolm, Y. Rathi, and C.-F. Westin. “Processing and visualization of diffusion MRI.” In T. Deserno, editor, *Recent Advances in Biomedical Image Processing and Analysis*, chapter 16, pages 387–410. Springer, 2011.

neural fibers

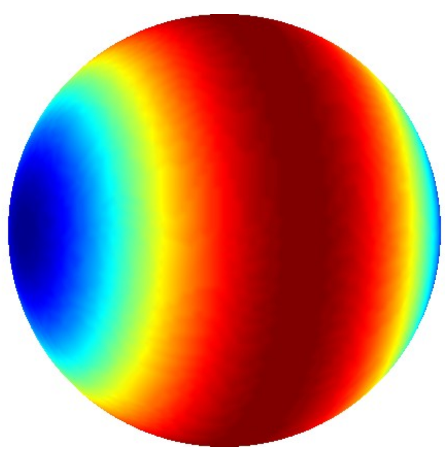




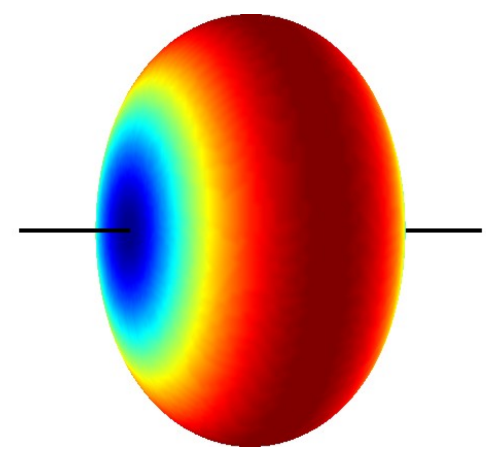
[Williams 97]



[Huettel 2004]



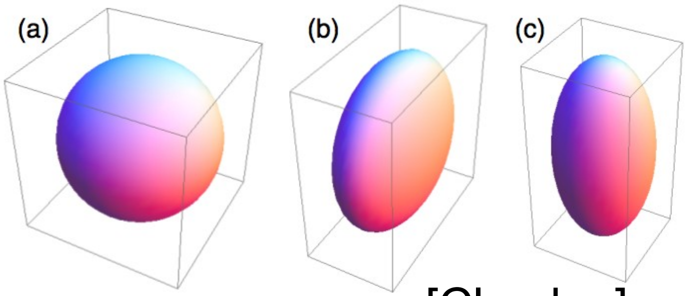
isotropic scanner signal



directed signal

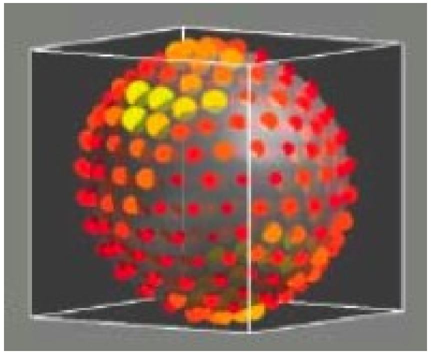
imaging techniques

- Diffusion tensor imaging (DTI)
  - single-tensor
  - [Basser 94]



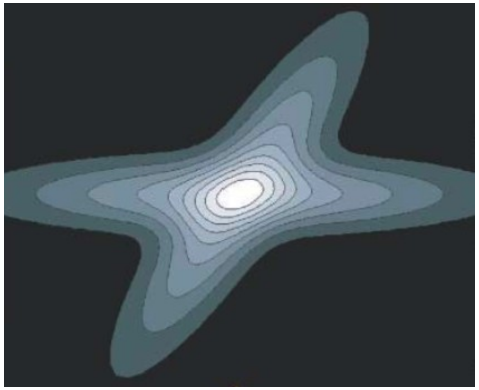
[Clayden]

- High Angular Resolution (QBI)
  - multi-fiber
  - [Tuch 02]

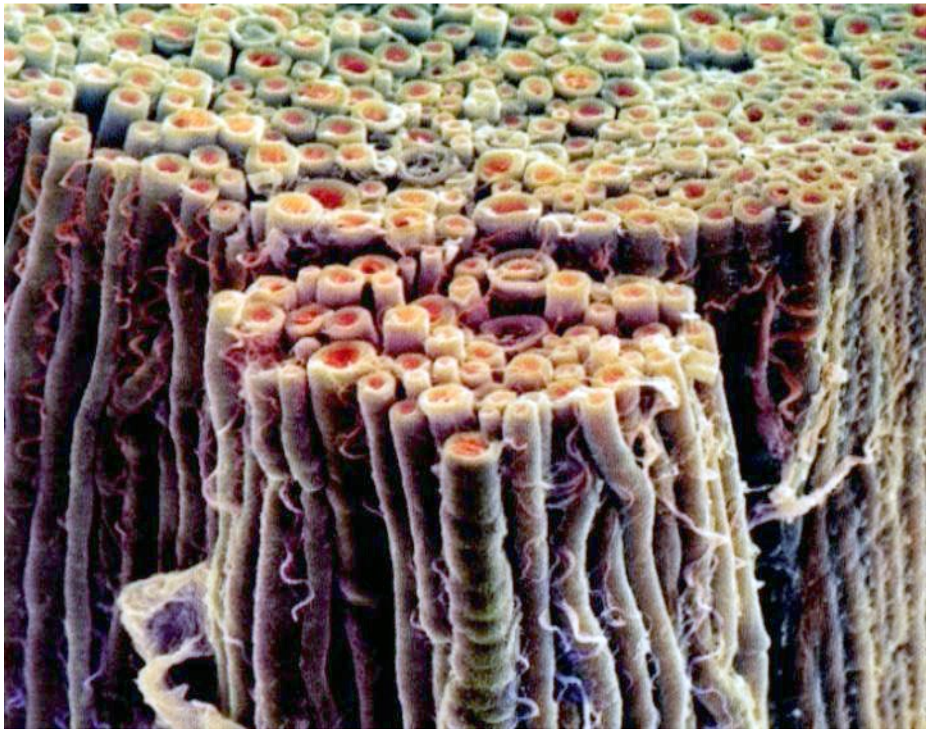


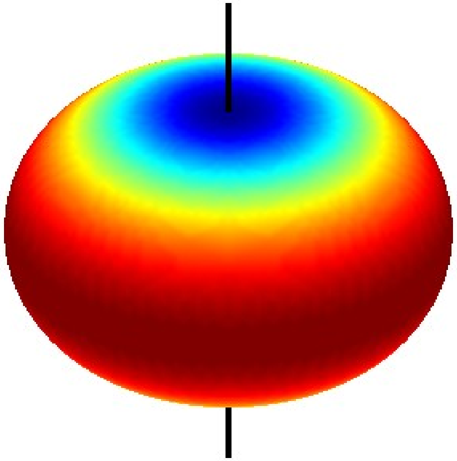
[Tuch]

- Diffusion Spectrum (DSI)
  - radial information
  - [Wedeen 05, Hagmann 05]



[Hagmann]



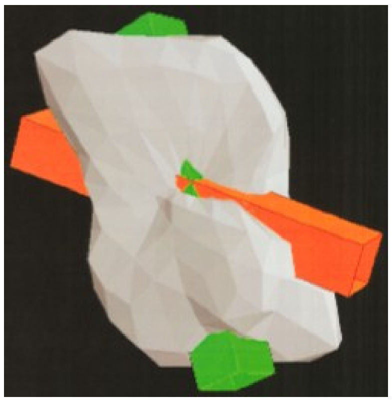


one neural fiber
0.2 to 20 microns

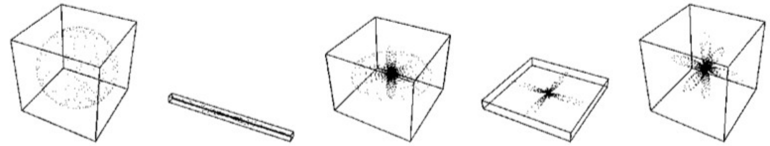
one scanned voxel
1.7 x 1.7 x 1.7 mm

George Bartzokis, UCLA

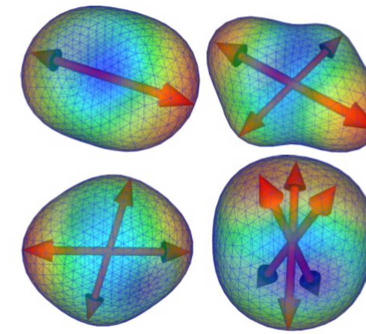
voxel reconstruction



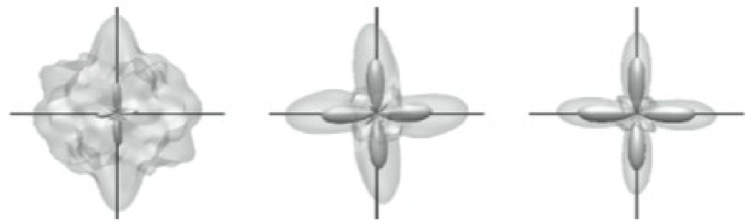
tensor mixtures [Tuch 02]



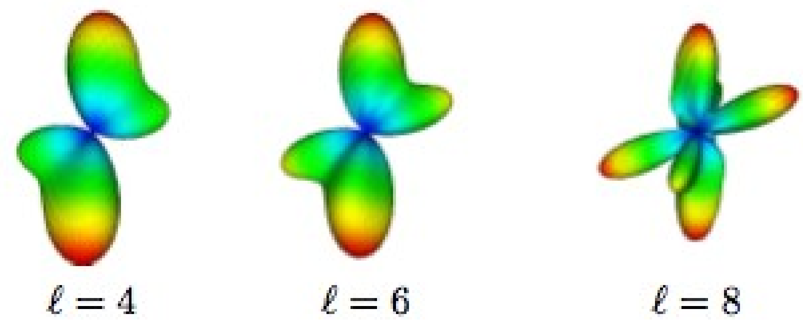
PAS [Jansons 03], MESD [Alexander 05]



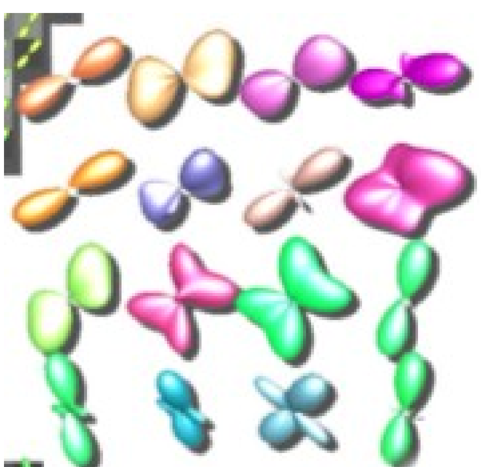
HOT [Hlawitschka 05]



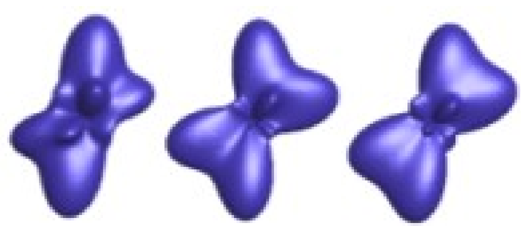
SD [Tournier 04]



spherical harmonics [Descoteaux 07]



VMF [Kumar 08]

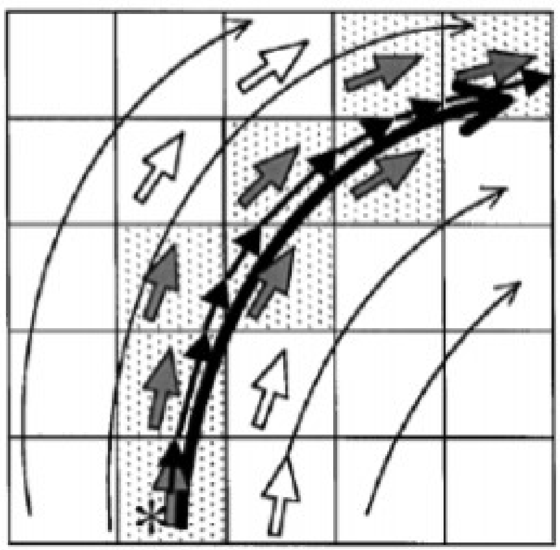


DOT [Ozarslan 05]

Wishart [Jian 07]

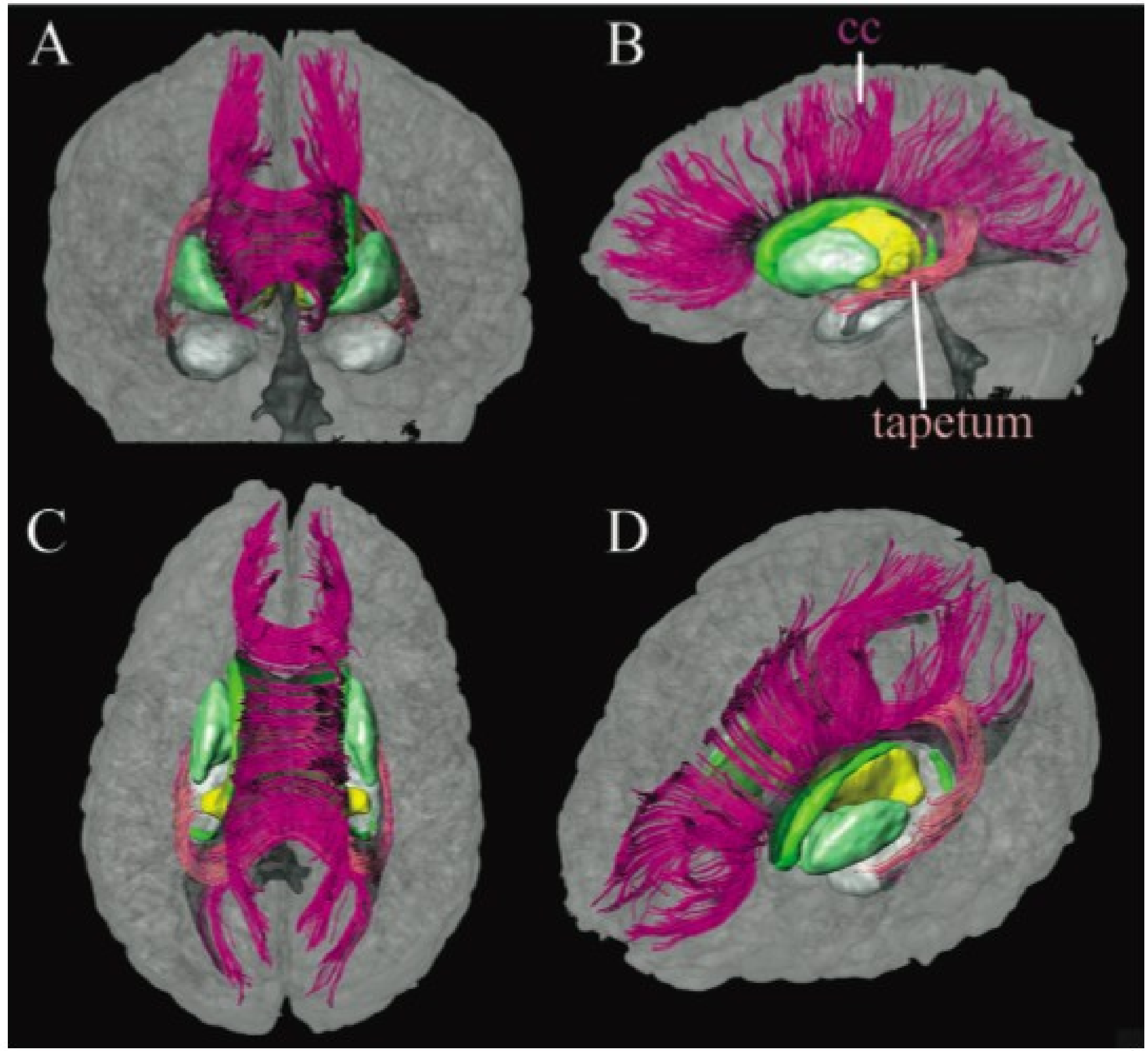
deterministic tractography

streamlin
e



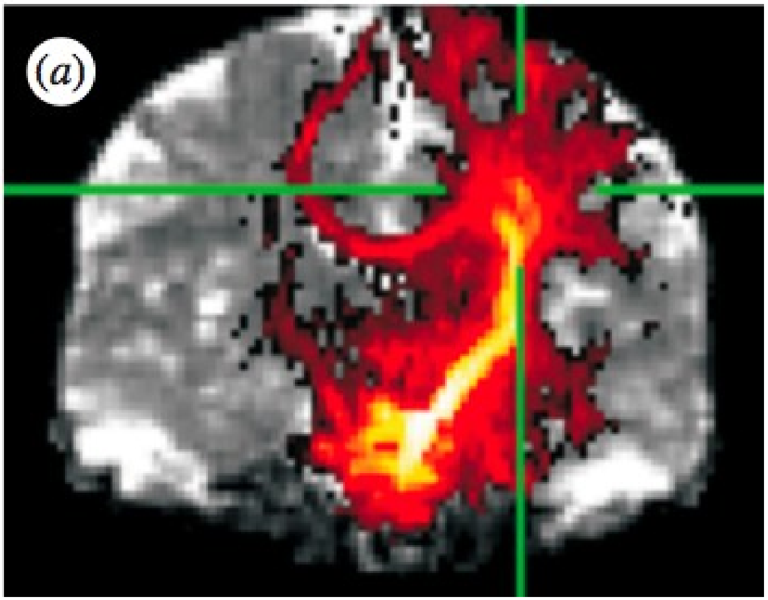
[Mori 99]

[Conturo 99, Basser 00]

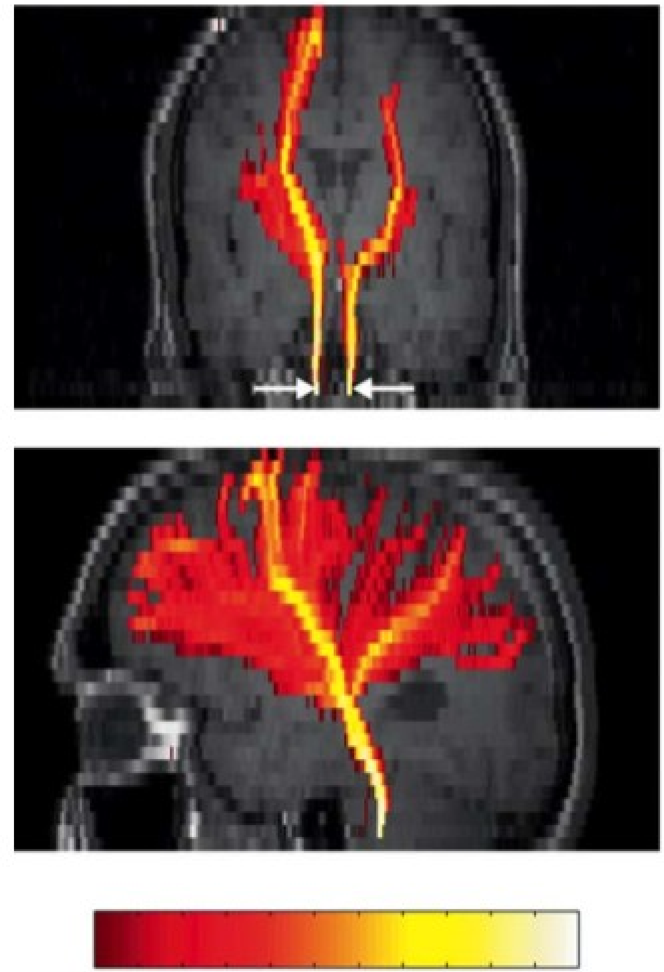


[Wakana 03]

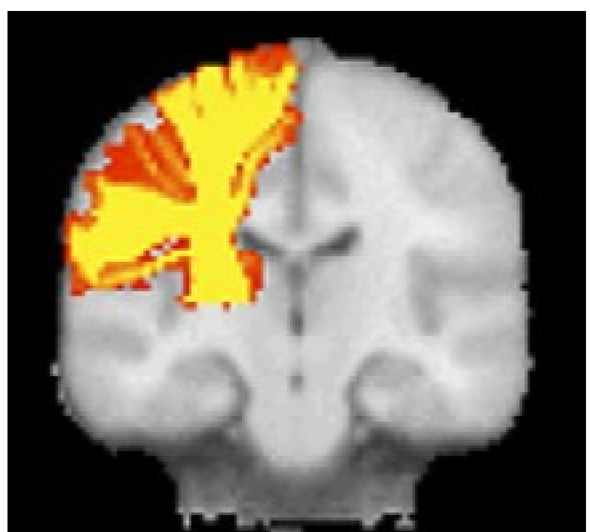
probabilistic tractography



[Parker 05]

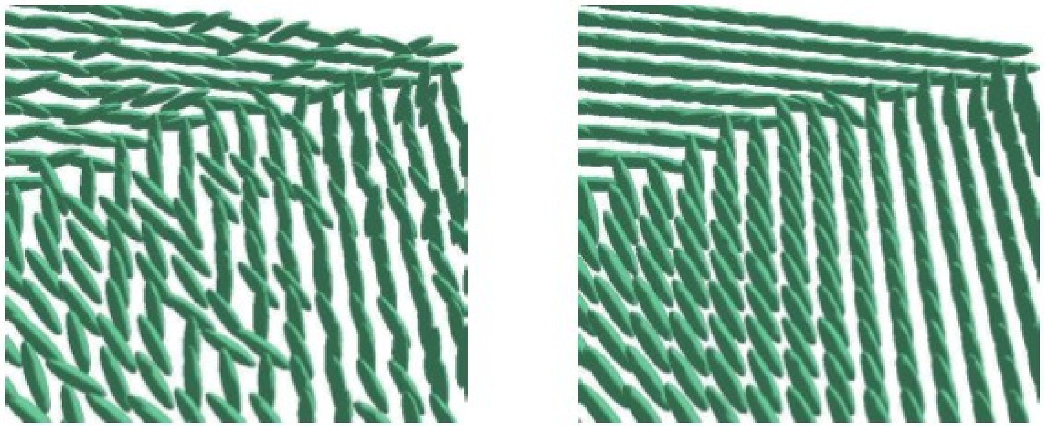


[Friman 06]



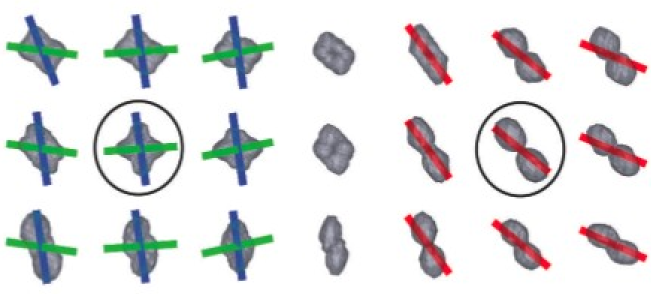
[Behrens 07]

spatial regularization

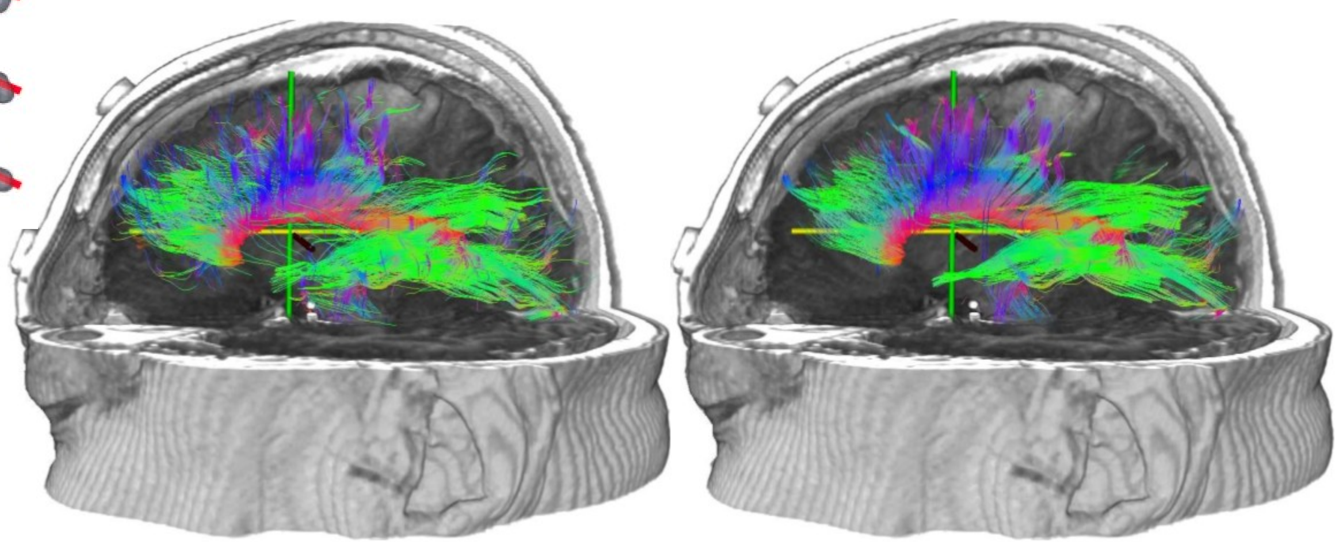


[Tschumperle 03]

pre/post-processing



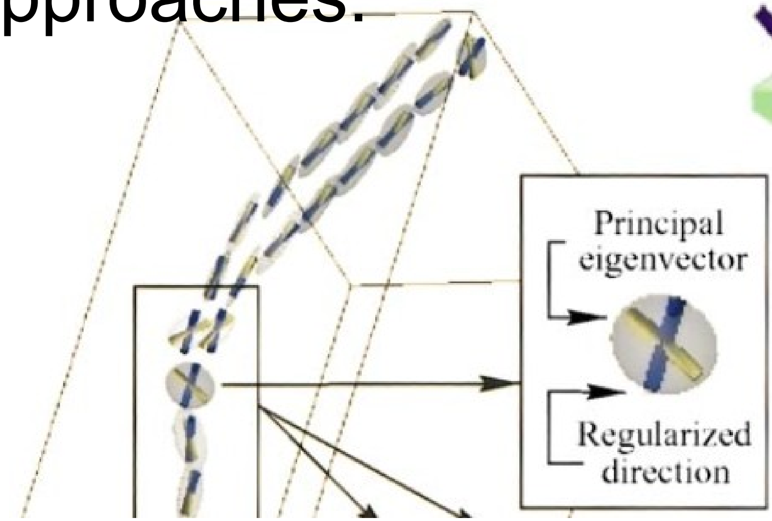
[Savadjiev 07]



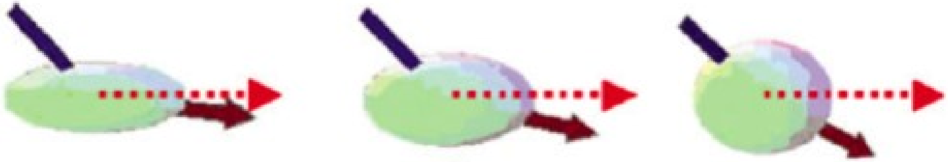
[Fillard 07]

tract regularization

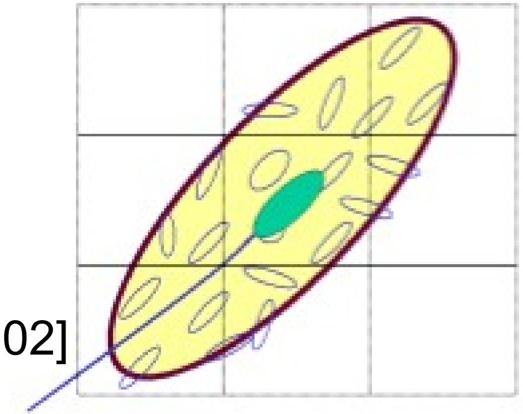
deterministic approaches:



spaghetti [Poupon 00]



deflection [Lazar 03]



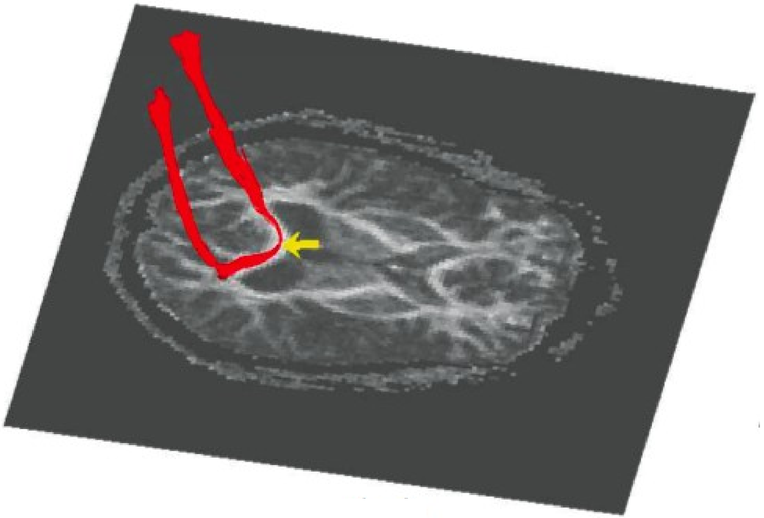
[Zhukov 02]

single-tensor path regularization

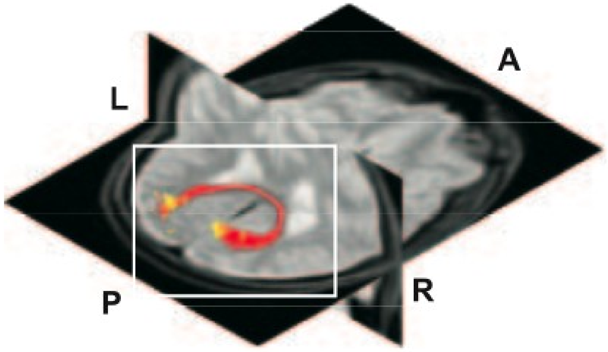
tract regularization

probabilistic
approaches:

linear Kalman filter [Gossl 02]



particle filtering [Zhang 07]



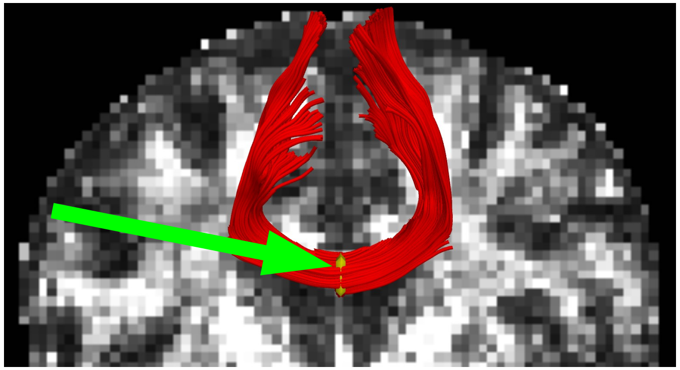
stochastic [Bjornemo 02]

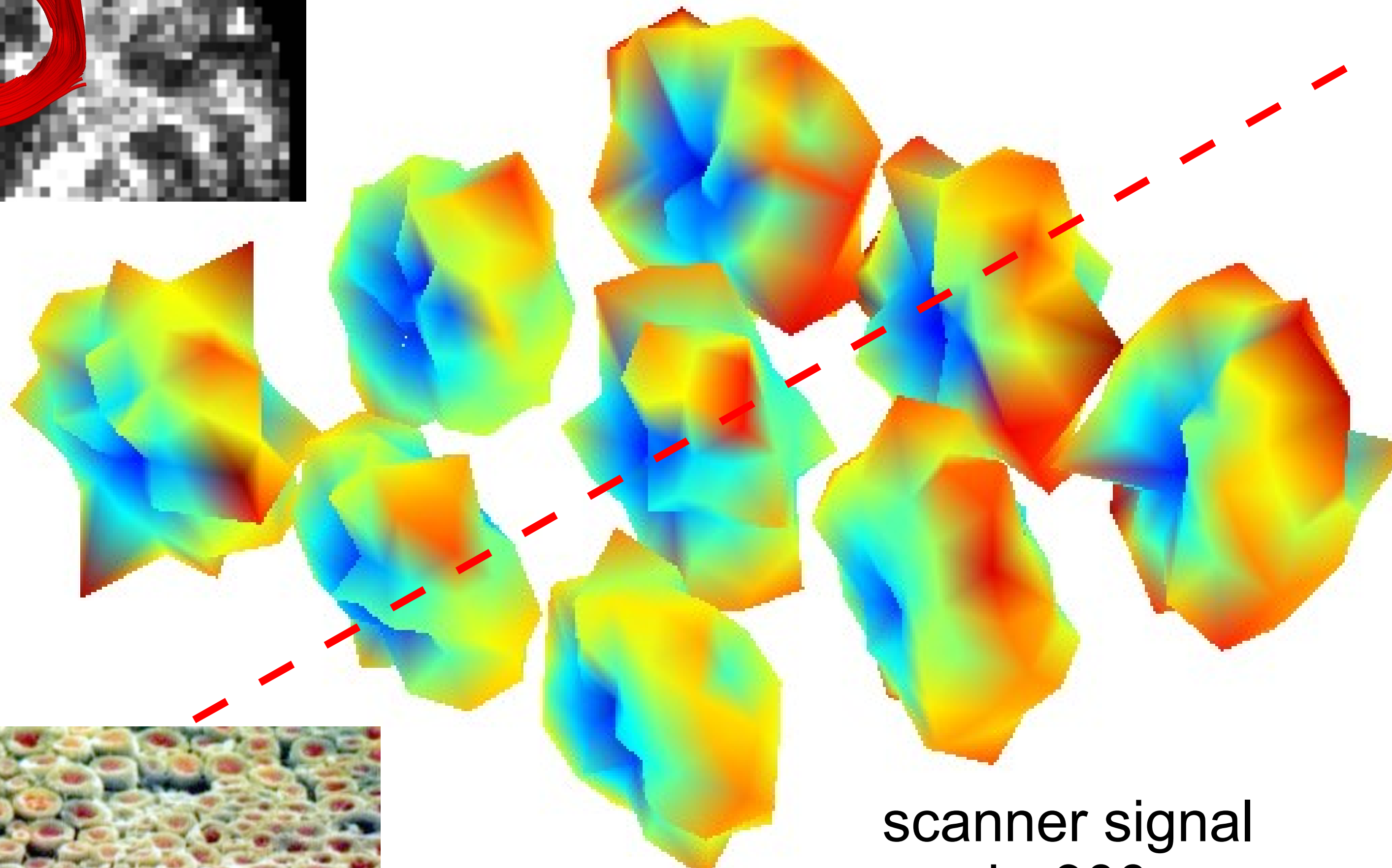
single-tensor
path
regularization

method:
the model

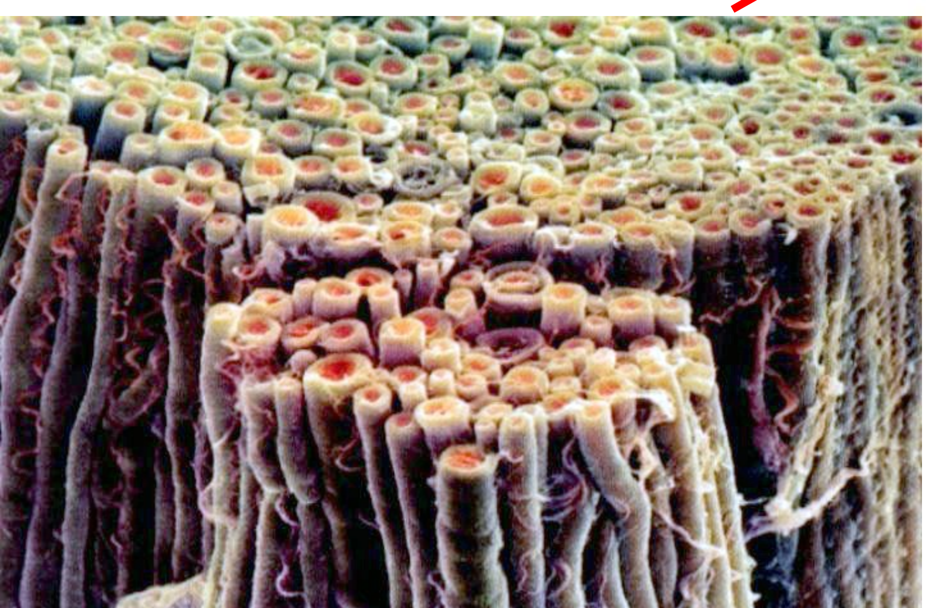
Y. Rathi, J. G. Malcolm, O. Michailovich, C.-F. Westin, M. E. Shenton, and S. Bouix. Tensor-kernels for simultaneous fiber model estimation and tractography. *Magnetic Resonance in Medicine*, 64(1):138–148, 2010.

J. G. Malcolm, M. E. Shenton, and Y. Rathi. Neural tractography using an unscented Kalman filter. In *Information Processing in Medical Imaging (IPMI)*, pages 126–138, 2009.





scanner signal
b=900
51 directions
1.7mm isotropic voxel
17 minute scan



[Bartzokis]

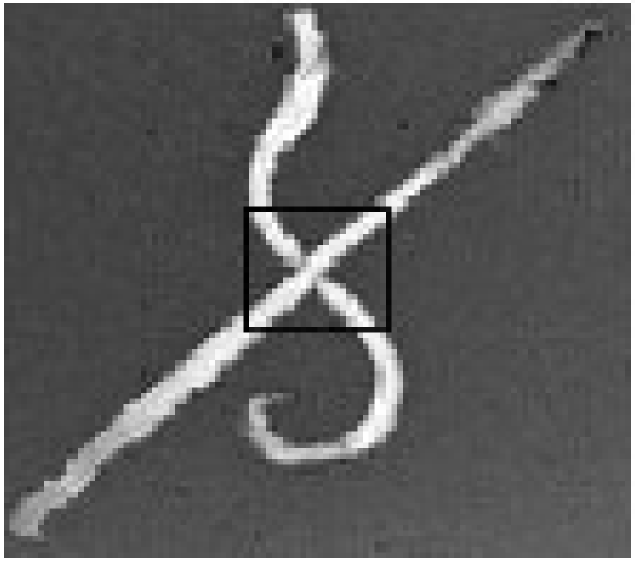
DTI

$$S(\boldsymbol{u}) = s_0 e^{-b\boldsymbol{u}^T D \boldsymbol{u}}$$

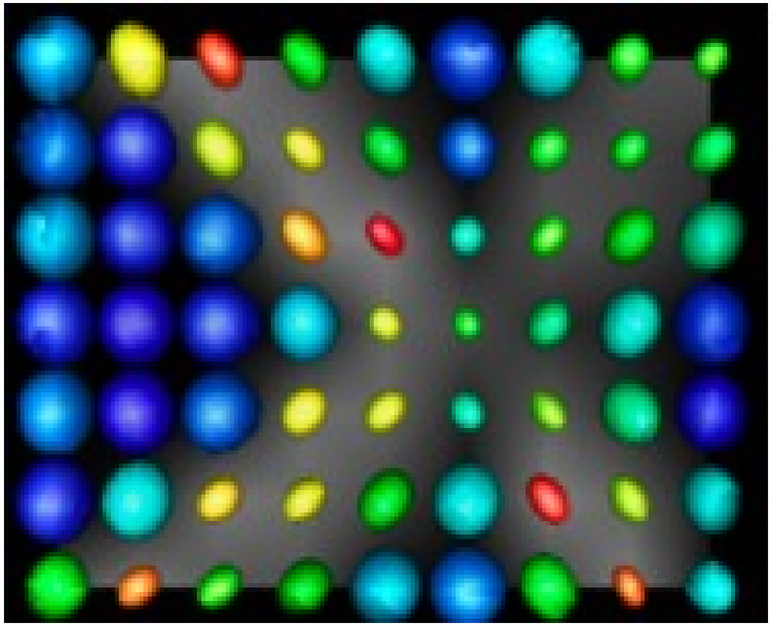
D	diffusion tensor	6 parameters
$\boldsymbol{u}$	unit direction	
b	acquisition constant	
s_0	null signal (b=0)	

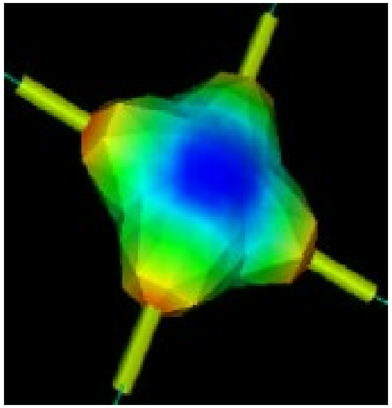
[Basser 94]

single-tensor limitations



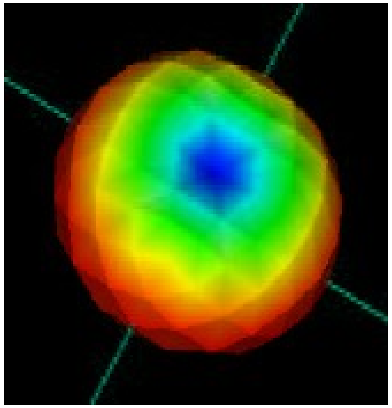
rat spinal nerves





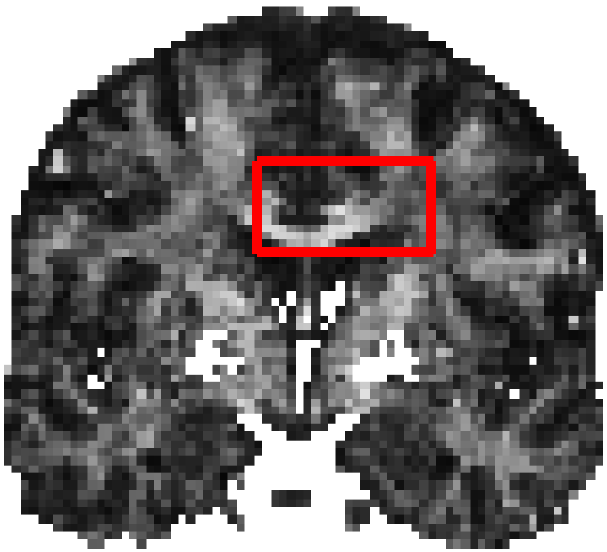
true ODF

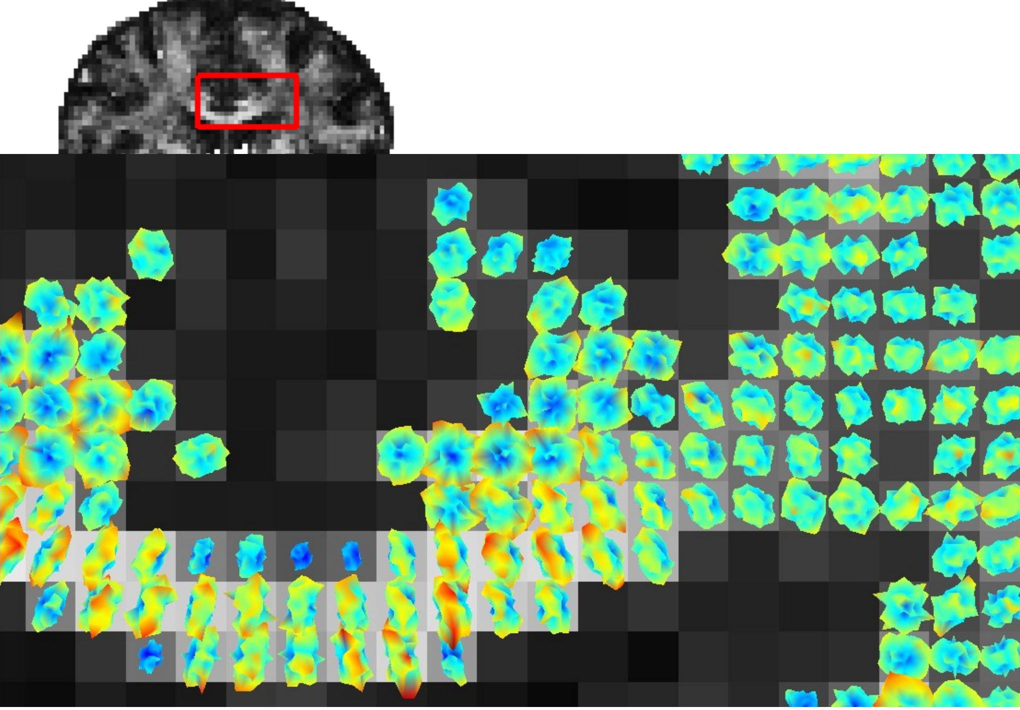
[Campbell, Descoteaux]



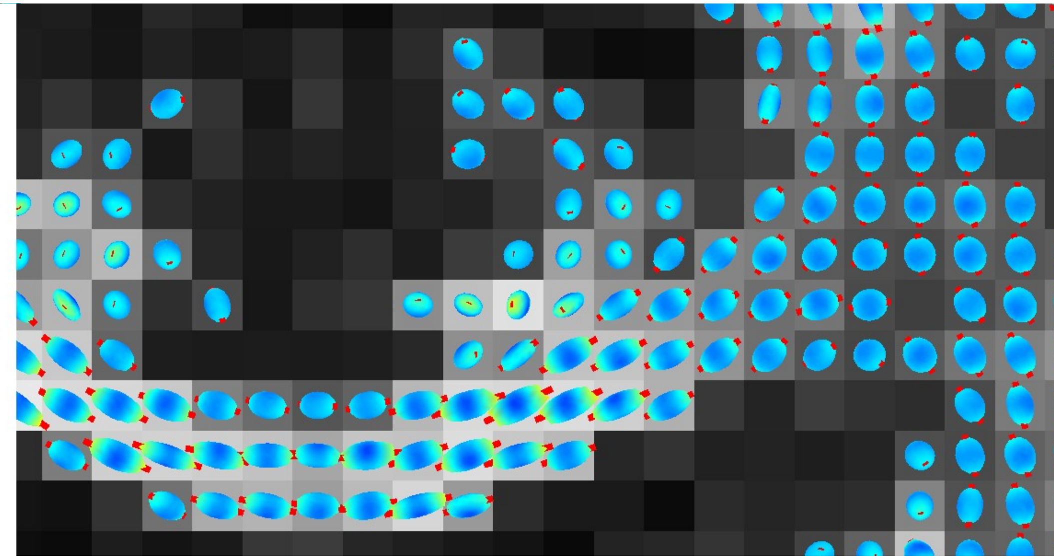
single-tensor ODF

[Descoteaux]

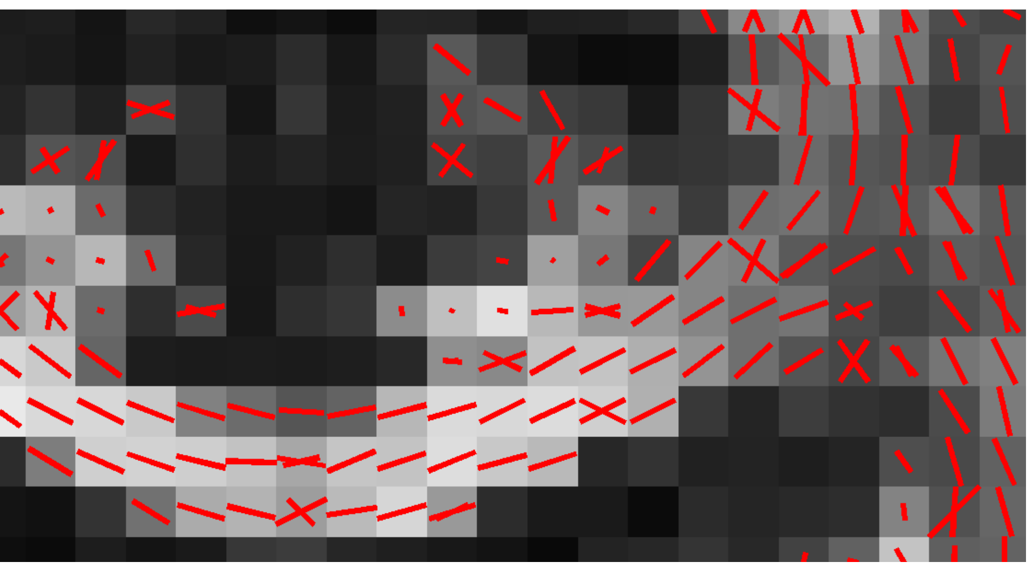




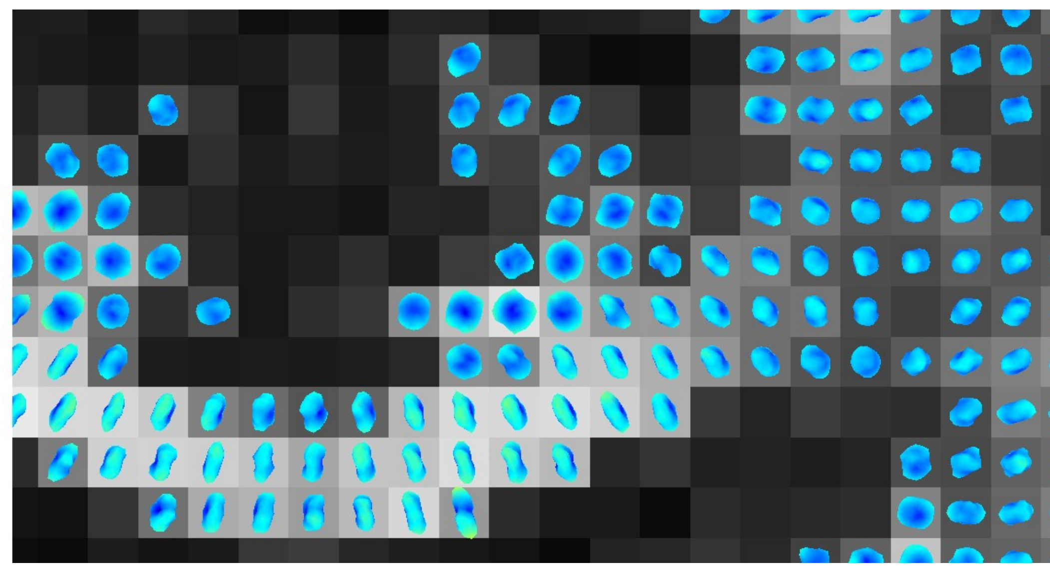
raw dMRI signal



single-tensor ODF



two-tensor axis



spherical harmonic ODF

multi-tensor signal model

$$S(\boldsymbol{u}) = s_0 \sum_j w_j e^{-b\boldsymbol{u}^T D_j \boldsymbol{u}}$$

D_j	diffusion tensor
$\boldsymbol{u}$	unit direction
w_j	convex weights
b	acquisition constant
s_0	null signal (b=0)

model assumptions

...for now

Two fibers

Fixed volume fractions

Tensors are elliptic or isotropic

model parameters

for two fibers...

...two principal directions $\boldsymbol{m} \in \mathbb{R}^3$

...two primary eigenvalues $\lambda_1 \in \mathbb{R}$

...two minor eigenvalues $\lambda_2 \in \mathbb{R}$

5 + 5 = 10 parameters

model parameters

for two fibers...

...two principal directions $\boldsymbol{m} \in \mathbb{R}^3$

...two primary eigenvalues $\lambda_1 \in \mathbb{R}$

...two minor eigenvalues $\lambda_2 \in \mathbb{R}$

5 + 5 = 10 parameters

$$S(\boldsymbol{u}) = 0.5\, s_0\, e^{-b \boldsymbol{u}^T D_1 \boldsymbol{u}} + 0.5\, s_0\, e^{-b \boldsymbol{u}^T D_2 \boldsymbol{u}}$$

$$D_1 = \lambda_{11} \boldsymbol{m}_1 \boldsymbol{m}_1^T + \lambda_{21} (\boldsymbol{p}\,\boldsymbol{p}^T + \boldsymbol{q}\,\boldsymbol{q}^T)$$

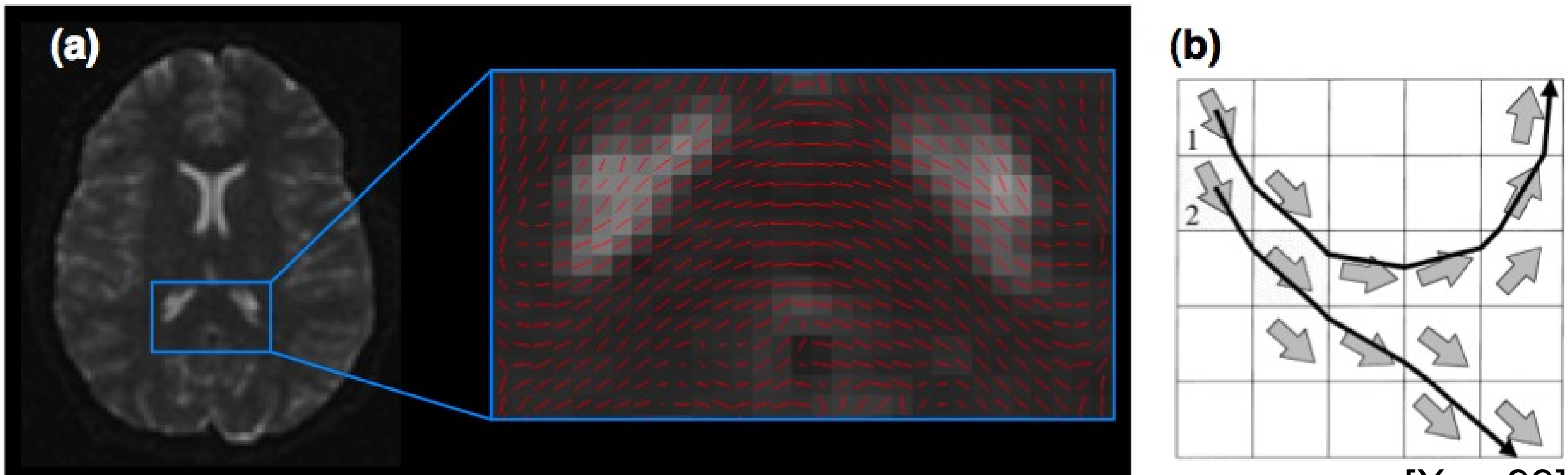
eigenvectors: $\boldsymbol{m}$, $\boldsymbol{p}$, $\boldsymbol{q}$

method:
estimating the model

J. G. Malcolm, O. Michailovich, S. Bouix, C.-F. Westin, M. E. Shenton, and Y. Rathi. A filtered approach to neural tractography using the Watson directional function. *Medical Image Analysis*, 14:58–69, 2010.

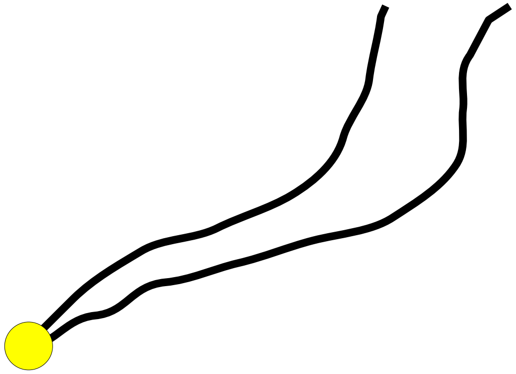
J. G. Malcolm, M. E. Shenton, and Y. Rathi. Filtered multi-tensor tractography. *IEEE Trans. on Medical Imaging*, 29:1664–1675, 2010.

independent estimation

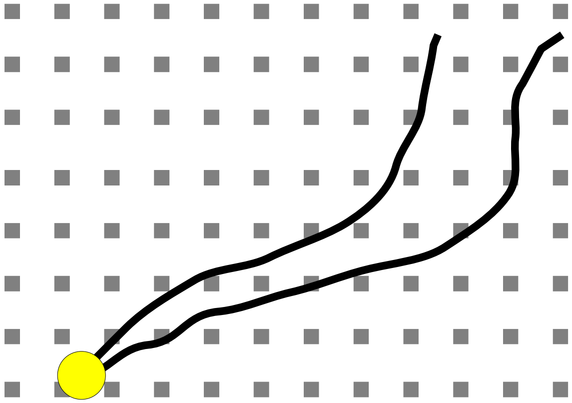


[Xue 99]

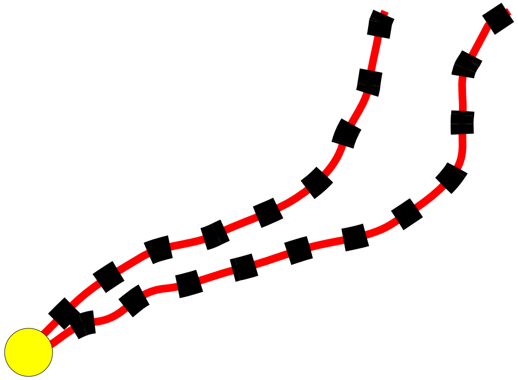
the system: a fiber



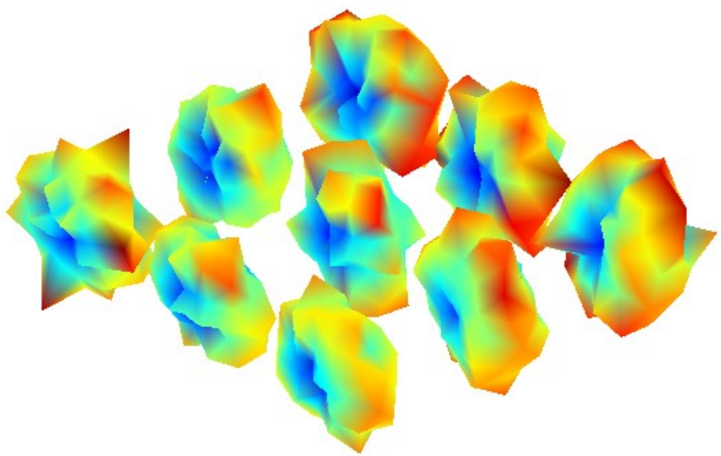
independent process



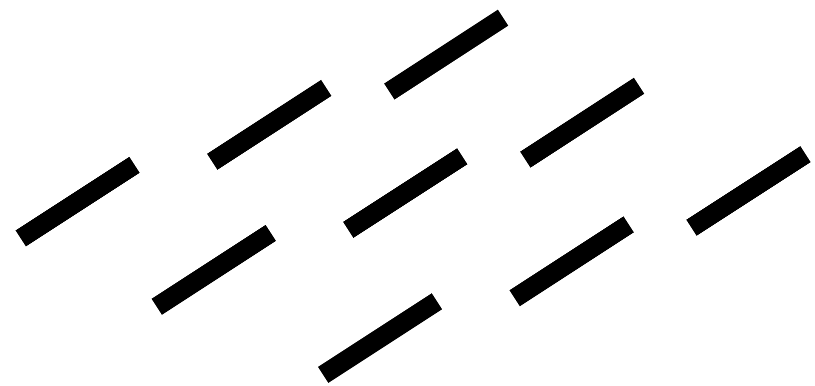
causal process



model-based filtering



scanner measurement



underlying model

objectives:

- estimate model from measurements
- suppress noise

notation

$\boldsymbol{x}_t$ state of system at time t

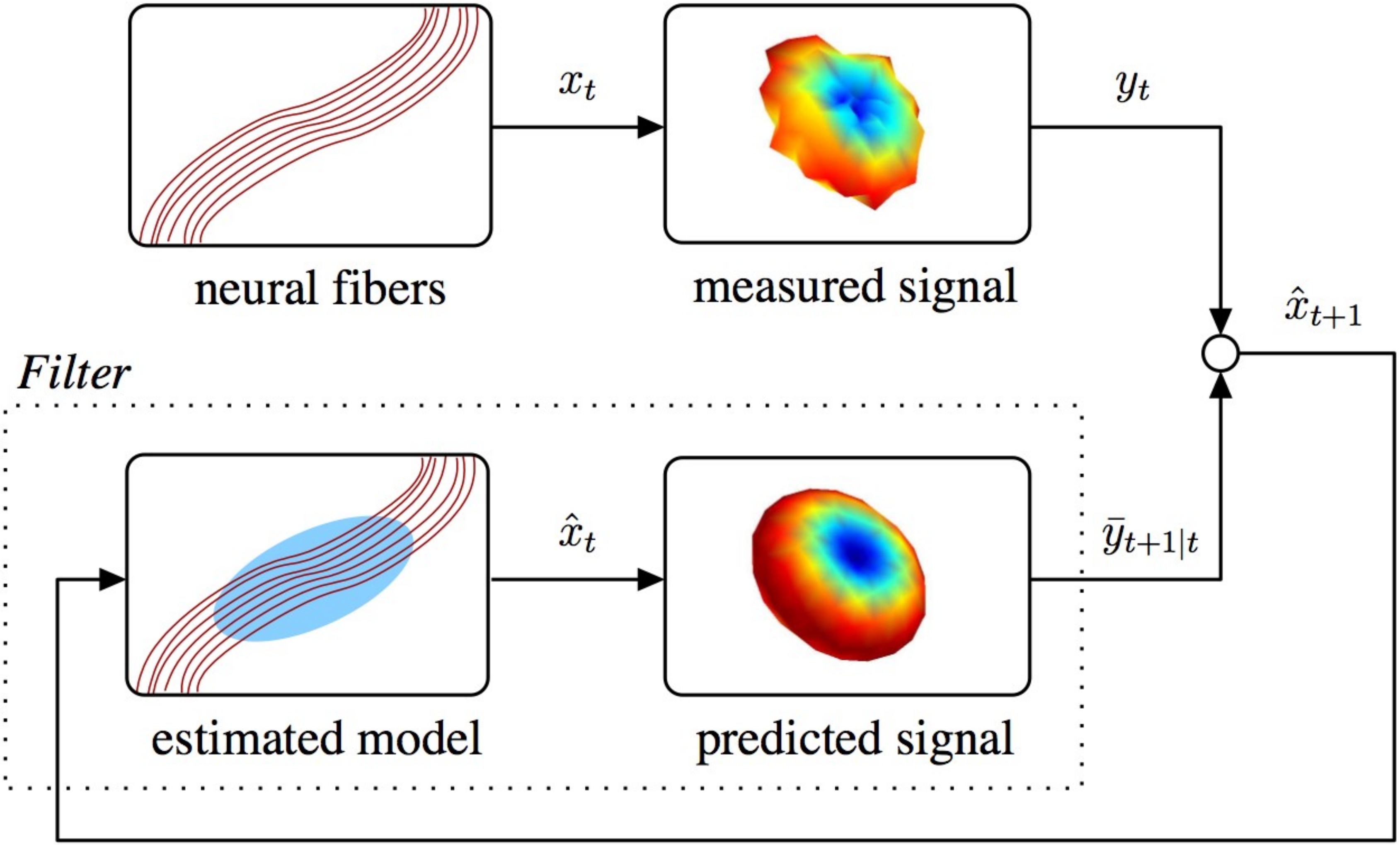
state = “model parameters”

$\boldsymbol{y}_t$ what you see at time t

observation, measurement

	linear	nonlinear
update:	$\boldsymbol{x}_{t+1} = F\,\boldsymbol{x}_t$	$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t)$
observation:	$\boldsymbol{y}_t = G\,\boldsymbol{x}_t$	$\boldsymbol{y}_t = g(\boldsymbol{x}_t)$

Kalman filtering



predict … measure … reconcile … repeat ...

$$\boldsymbol{x} = [\boldsymbol{m}_1 \lambda_{11} \lambda_{12} \boldsymbol{m}_2 \lambda_{21} \lambda_{22}]^T \in R^{10}$$

10 dimensional state

$\boldsymbol{y} \in R^m$ signal

$$\boldsymbol{x} = [\boldsymbol{m}_1 \lambda_{11} \lambda_{12} \boldsymbol{m}_2 \lambda_{21} \lambda_{22}]^T \in R^{10}$$

10 dimensional state

$$\boldsymbol{y} \in R^m$$ signal

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t) = \boldsymbol{x}_t$$

$$\boldsymbol{y}_t = g(\boldsymbol{x}_t) = S(\boldsymbol{u})$$

small steps
slowly varying
state

$$\boldsymbol{x} = [\boldsymbol{m}_1 \lambda_{11} \lambda_{12} \boldsymbol{m}_2 \lambda_{21} \lambda_{22}]^T \in R^{10}$$

10 dimensional state

$$\boldsymbol{y} \in R^m$$ signal

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t) = \boldsymbol{x}_t$$

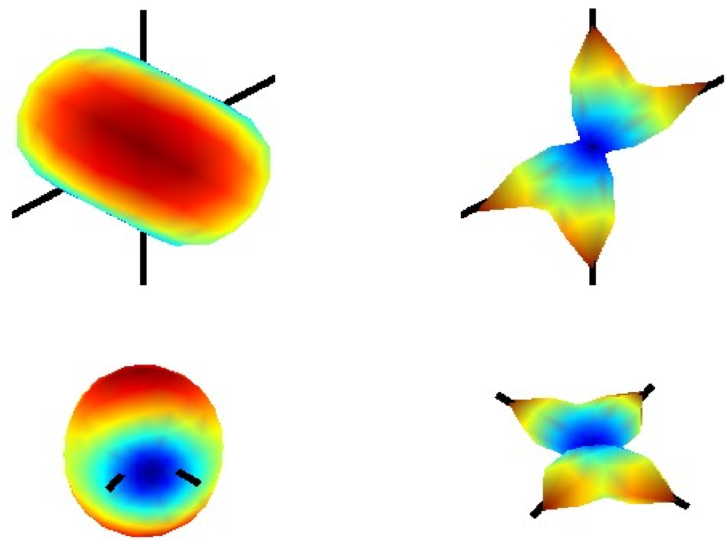
small steps slowly varying state

$$\boldsymbol{y}_t = g(\boldsymbol{x}_t) = S(\boldsymbol{u})$$

$$y(\boldsymbol{u}) = S(\boldsymbol{u}) = 0.5\, s_0 e^{-b\boldsymbol{u}^T D_1 \boldsymbol{u}} + 0.5\, s_0 e^{-b\boldsymbol{u}^T D_2 \boldsymbol{u}}$$

$$D = \lambda_1 \boldsymbol{m}\boldsymbol{m}^T + \lambda_2 (\boldsymbol{p}\boldsymbol{p}^T + \boldsymbol{q}\boldsymbol{q}^T)$$

Local fiber model: mixture of two tensors



diffusion MRI signal

orientation distribution function (ODF)

signal reconstruction is nonlinear

independent optimization

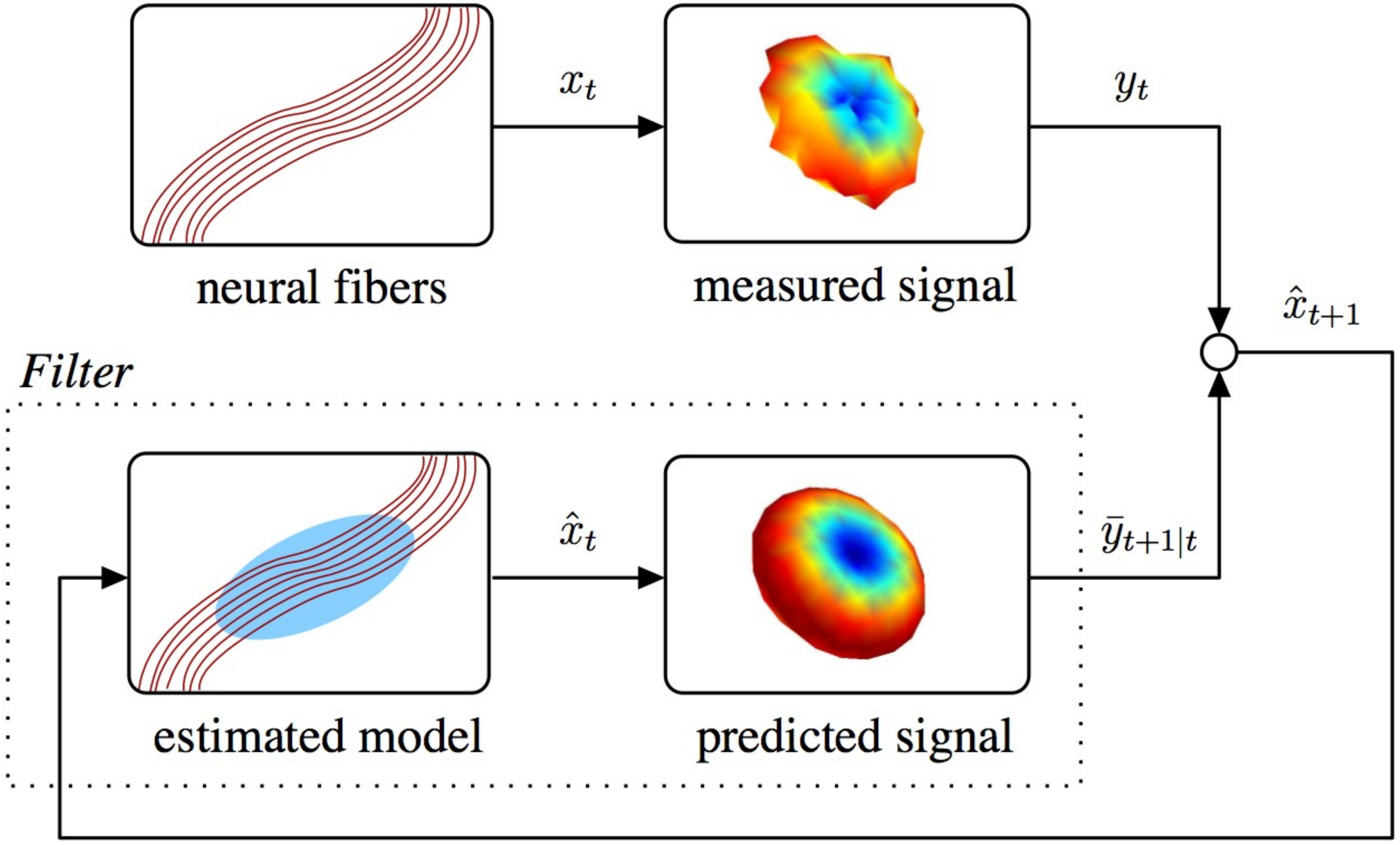
- least squares
  linearization
- gradient descent
  local minima
- Levenberg-Marquardt
  local minima

causal estimation

- extended Kalman filter
  mean + covariance
  linearization
- particle filter
  non-parametric
  sampling
- unscented Kalman filter
  mean + covariance
  no linearization
  limited sampling

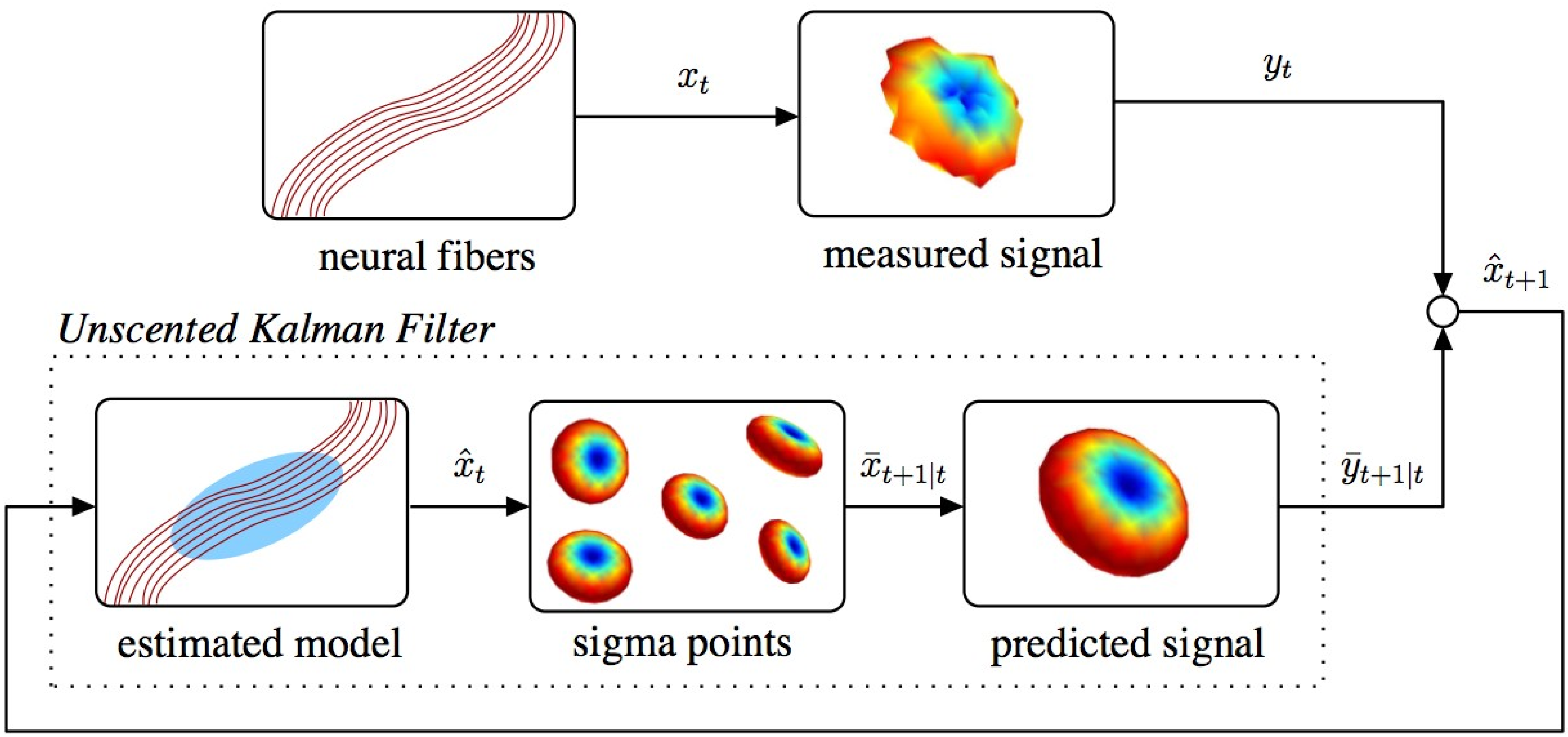
[Julier 04, Merwe 03]

linear Kalman filter



predict … measure … reconcile … repeat ...

unscented Kalman filter

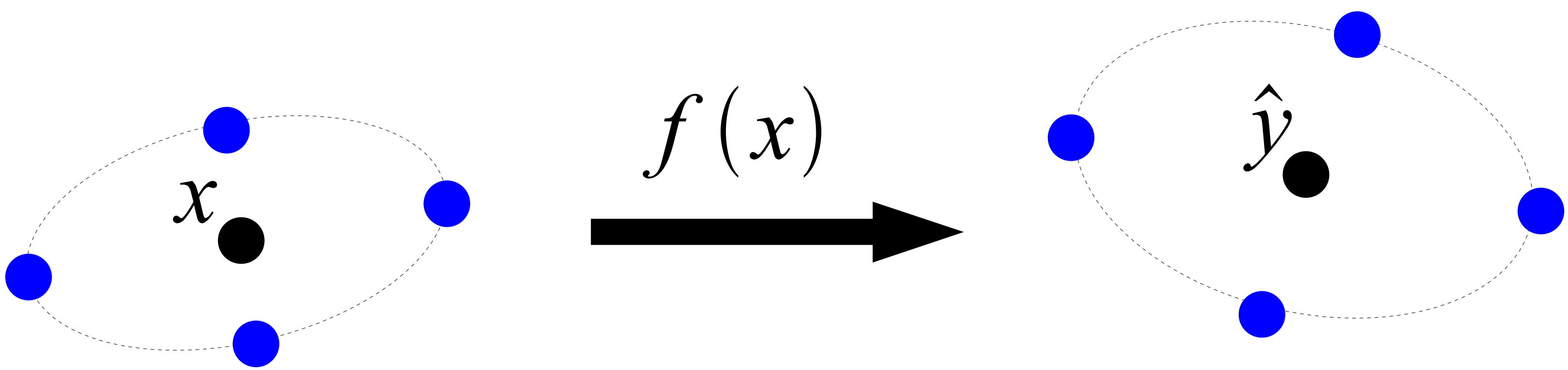


same update equations

modified prediction step

unscented transform

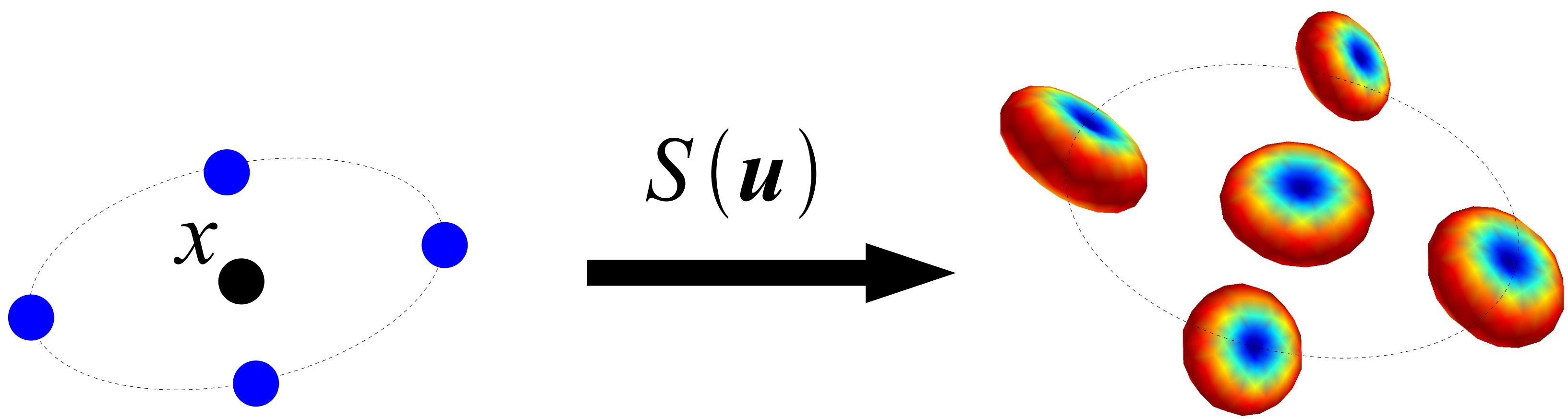
approximate the statistics...not the function



x, P_{xx} $\qquad\qquad$ $\hat{y}, \hat{P}_{yy}$

unscented transform

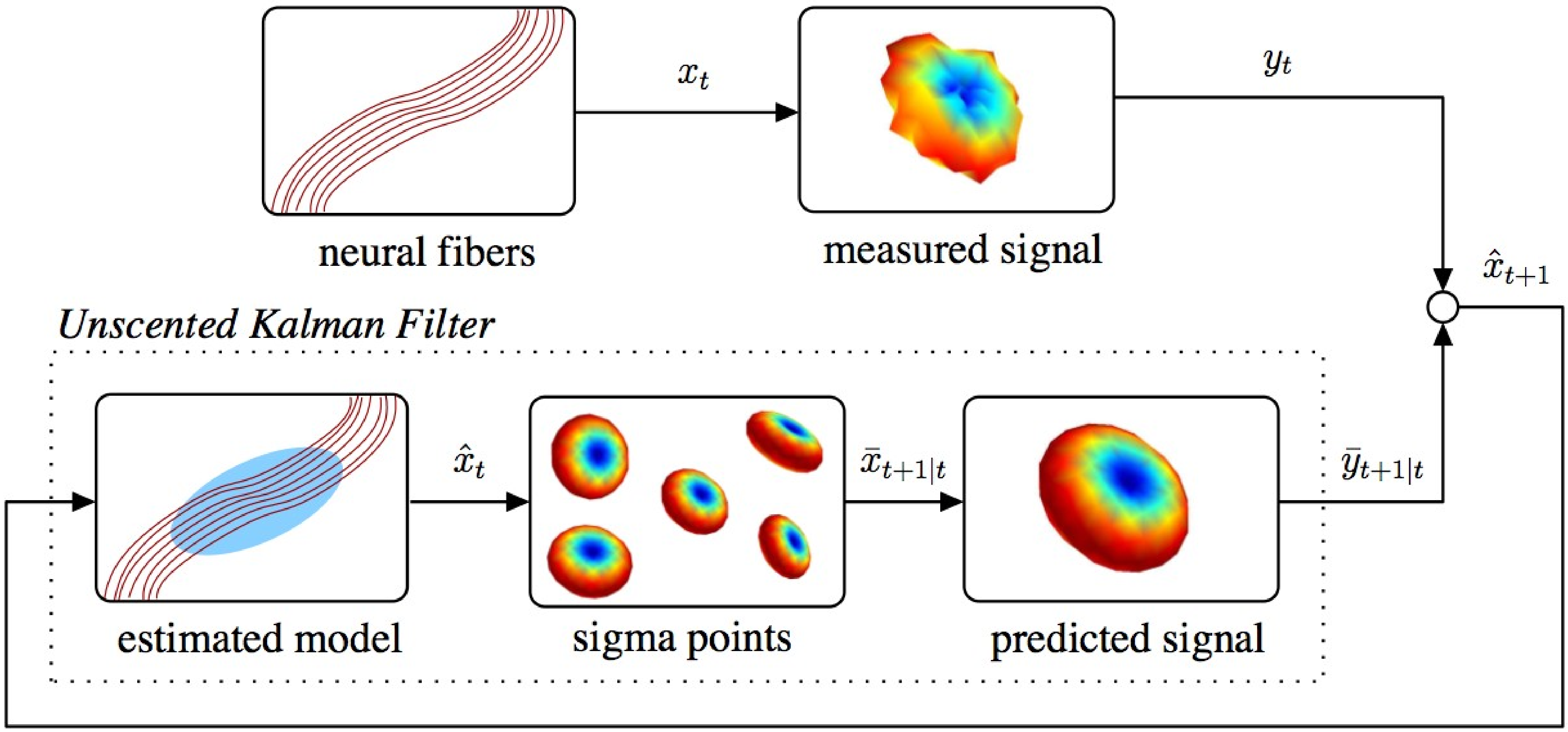
for signal reconstruction...



$x \,,\, P_{xx}$ $\qquad$ $\hat{y} \,,\, \hat{P}_{yy}$

$$\boldsymbol{x} = [\,\boldsymbol{m}_1\, k_1\, \boldsymbol{m}_2\, k_2\,]^T \in R^8$$

unscented Kalman filter

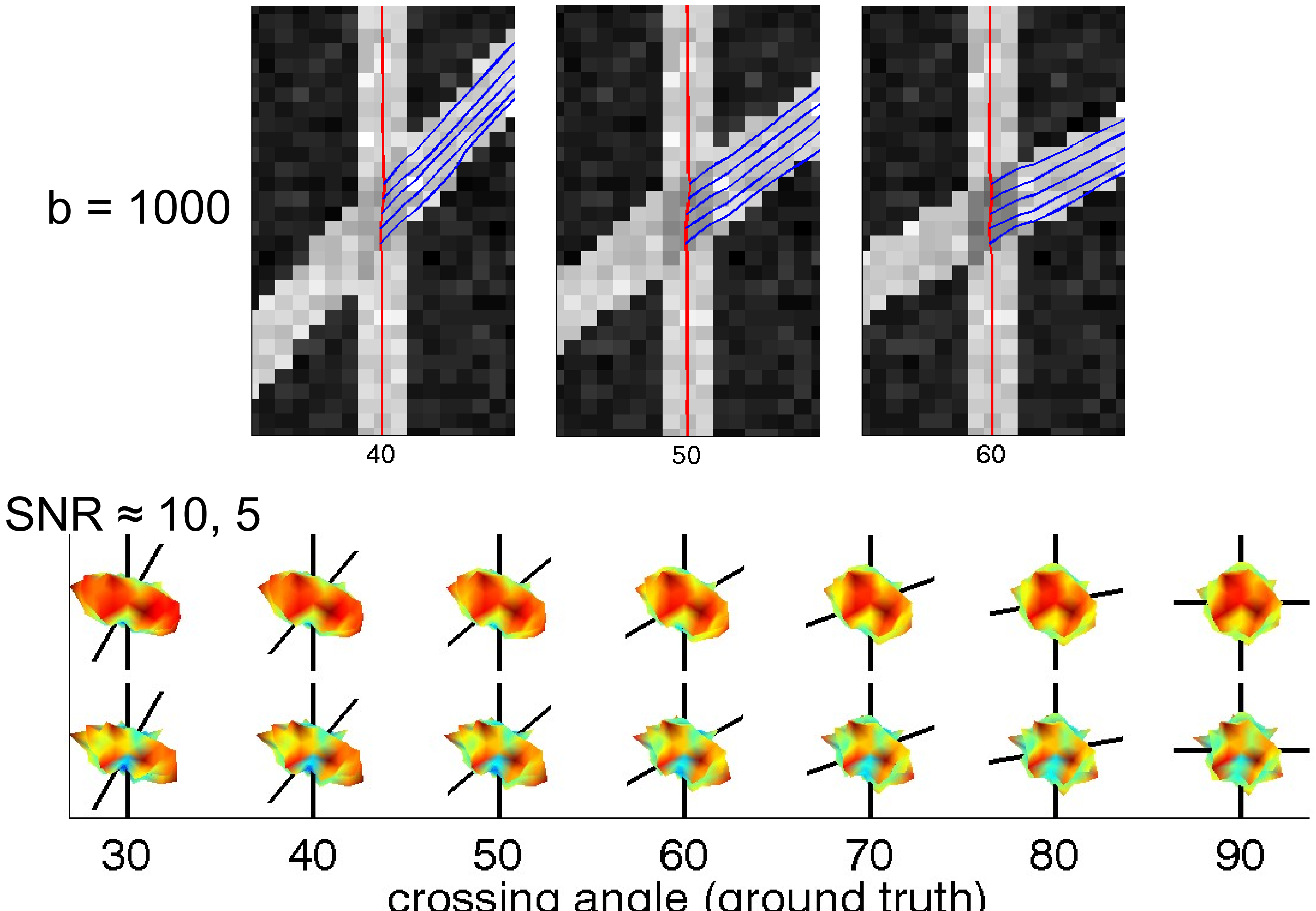


predict … measure … reconcile … repeat ...

diffusion tensor results

J. G. Malcolm, M. E. Shenton, and Y. Rathi. Filtered multi-tensor tractography. *IEEE Trans. on Medical Imaging*, 29:1664–1675, 2010.

synthetic validation


b = 1000
40
50
60
SNR ≈ 10, 5
30
40
50
60
70
80
90
crossing angle (ground truth)

nonlinear least squares

- Levenberg-Marquart
- two-tensor model
- initialize with ground truth (best possible)

spherical harmonics

- non-parametric
- order eight (8)
- fiber sharpening for peak detection (L=0.006)

[Descoteaux 07]

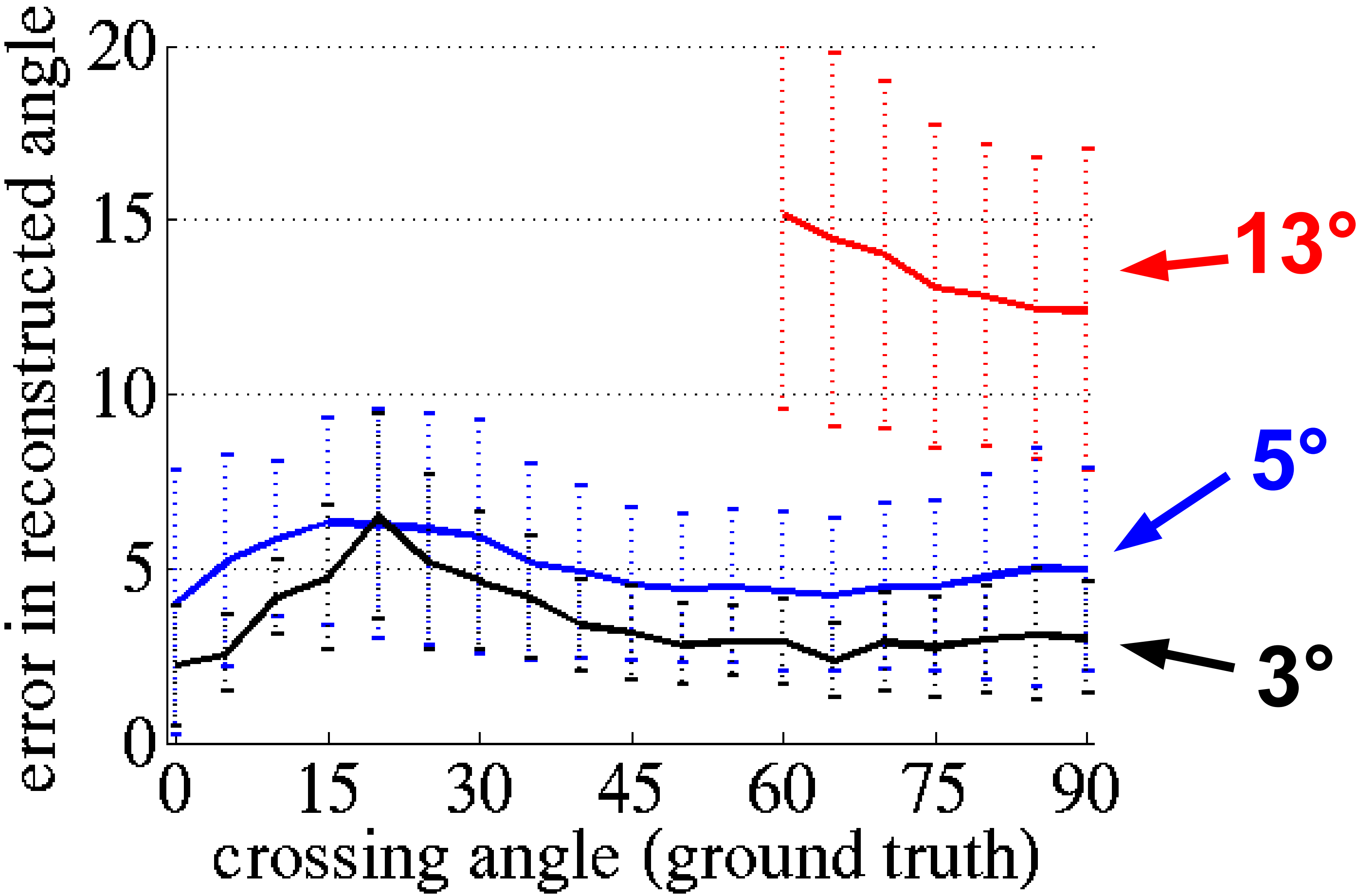
filtered tractography

- two-fiber model
- unscented Kalman filter

J. G. Malcolm, M. E. Shenton, and Y. Rathi. Neural tractography using an unscented Kalman filter. In *Information Processing in Medical Imaging (IPMI)*, pages 126–138, 2009.

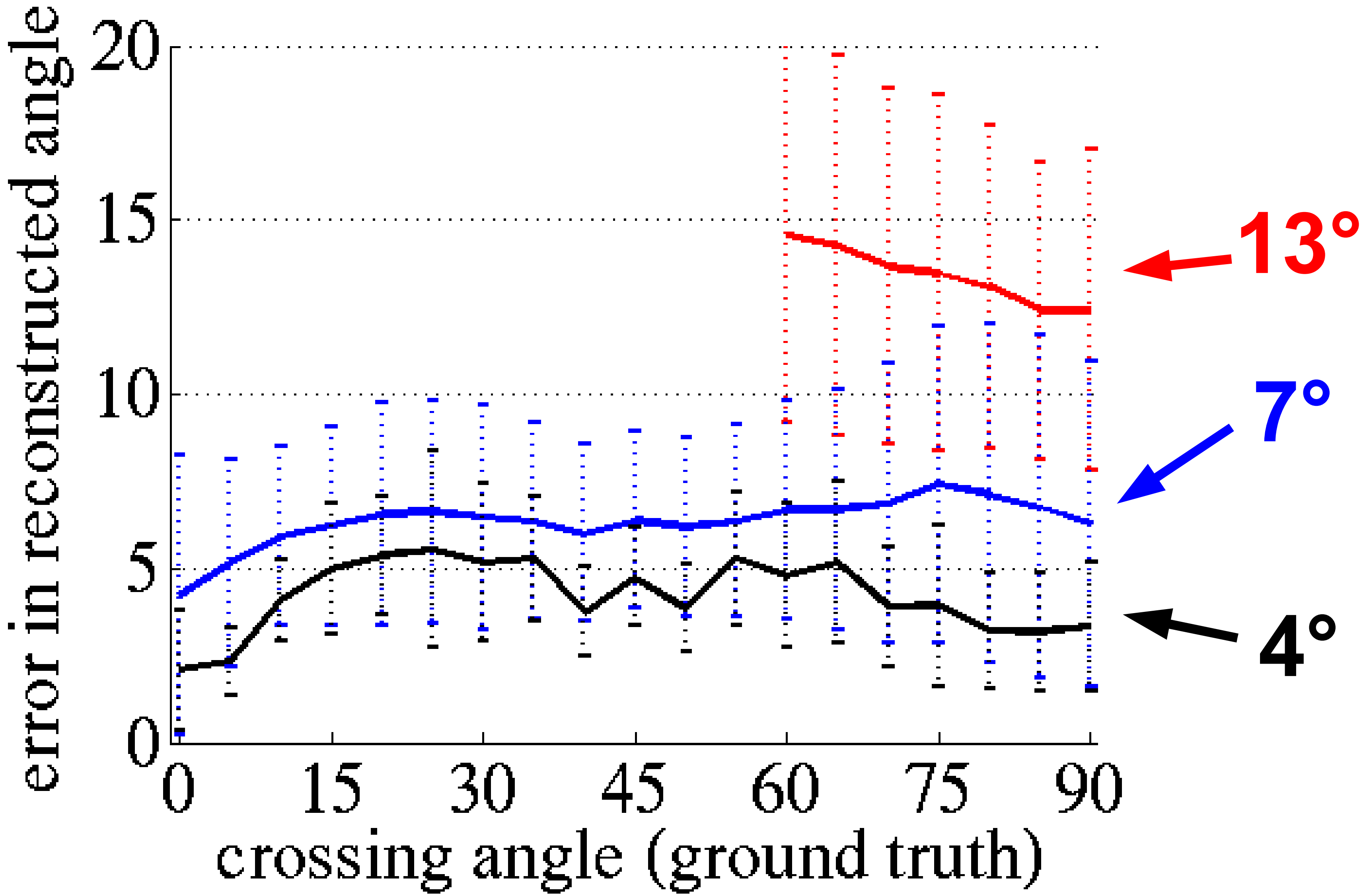
J. G. Malcolm, M. E. Shenton, and Y. Rathi. Filtered multi-tensor tractography. *IEEE Trans. on Medical Imaging*, 29:1664–1675, 2010.

angular reconstruction error



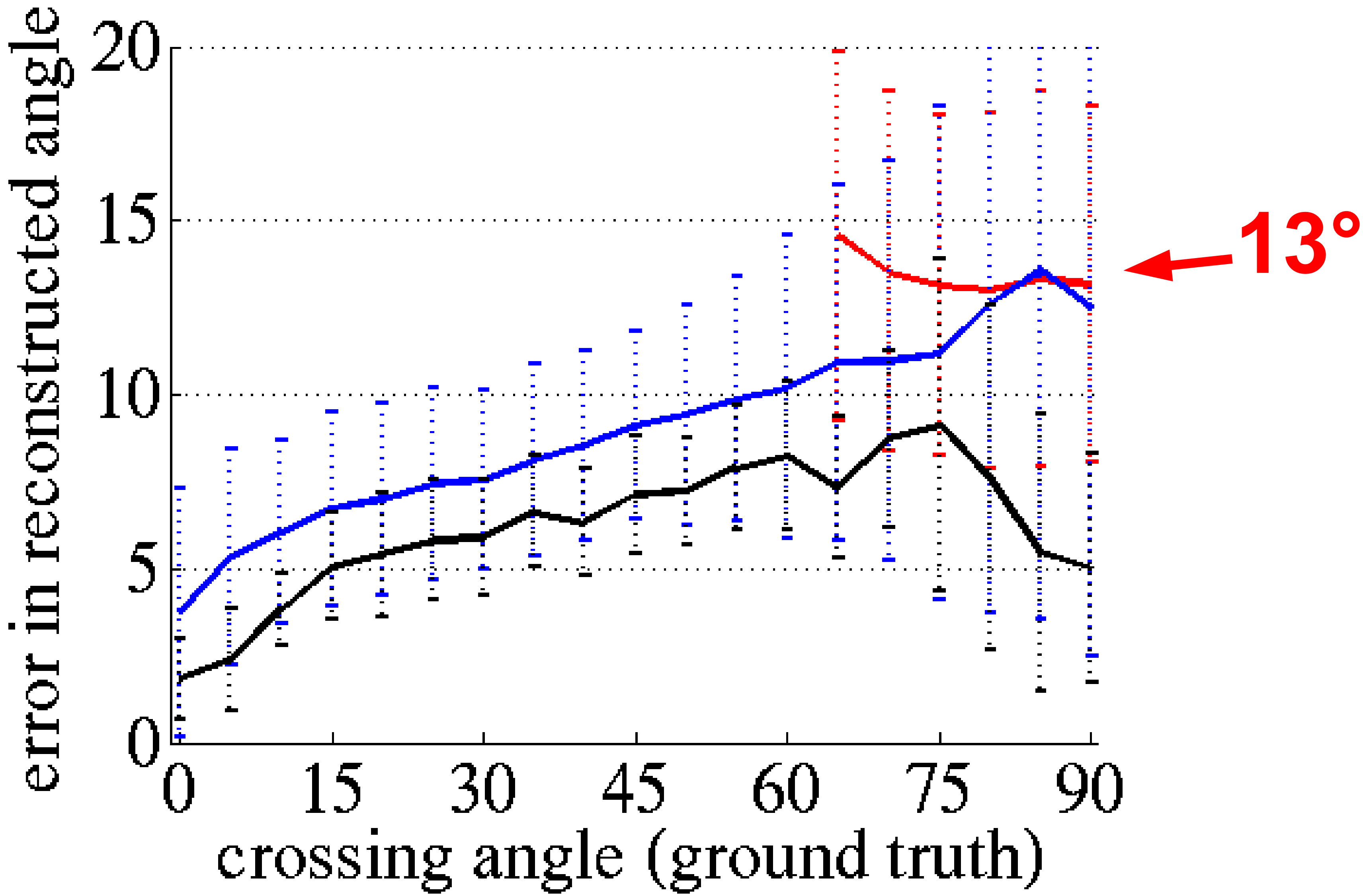
SNR ≈ 5, b = 1000

unequal signal field: 60/40



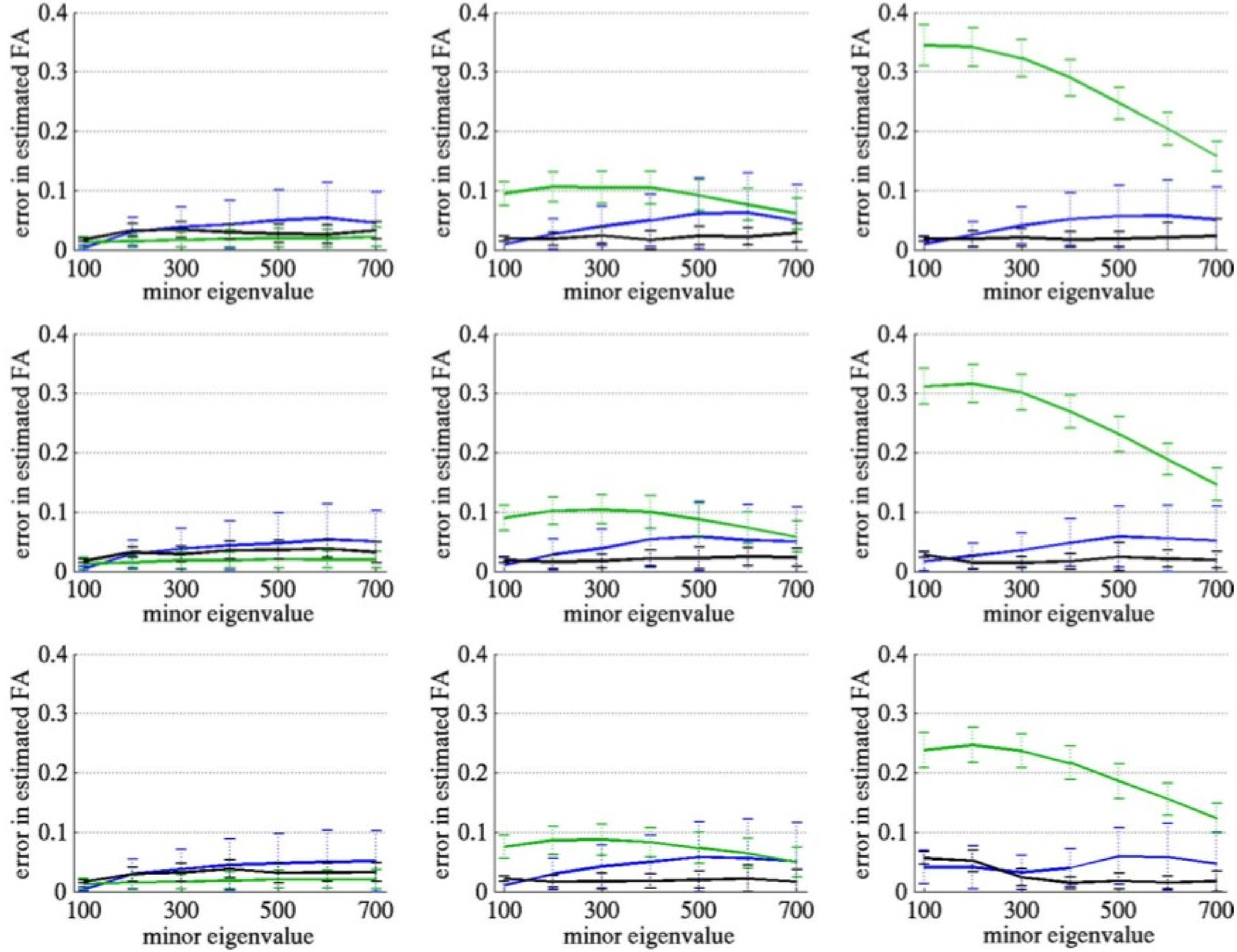
SNR ≈ 5, b = 1000

unequal signal field: 70/30



SNR ≈ 5, b = 1000

estimation fidelity



50/50

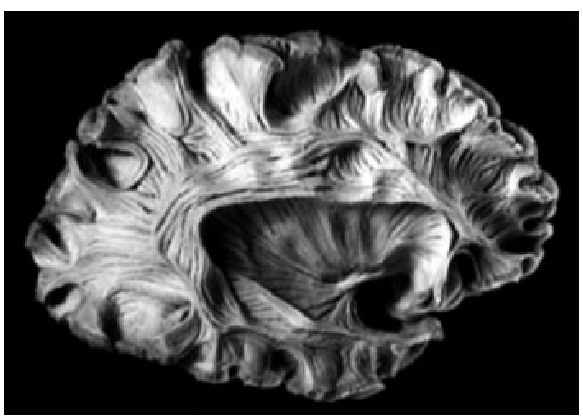
60/40

70/30

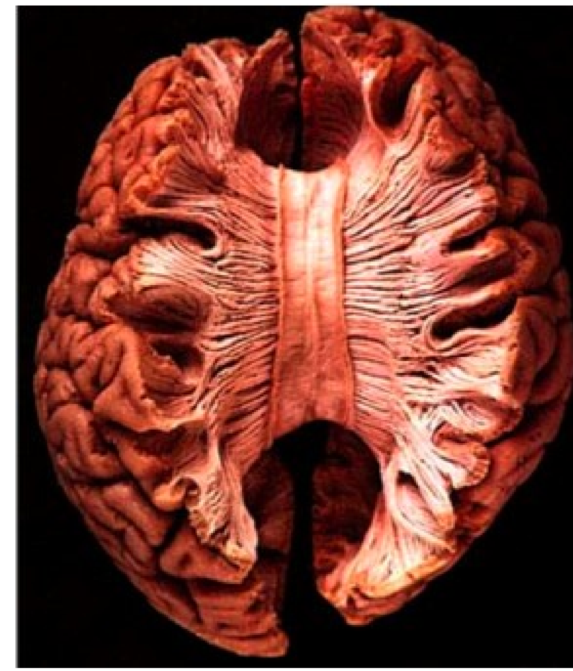
one fiber (no crossing)

45 degree crossing

90 degree crossing



in vivo



b=900
51 directions
1.7mm isotropic voxel
17 minute scan

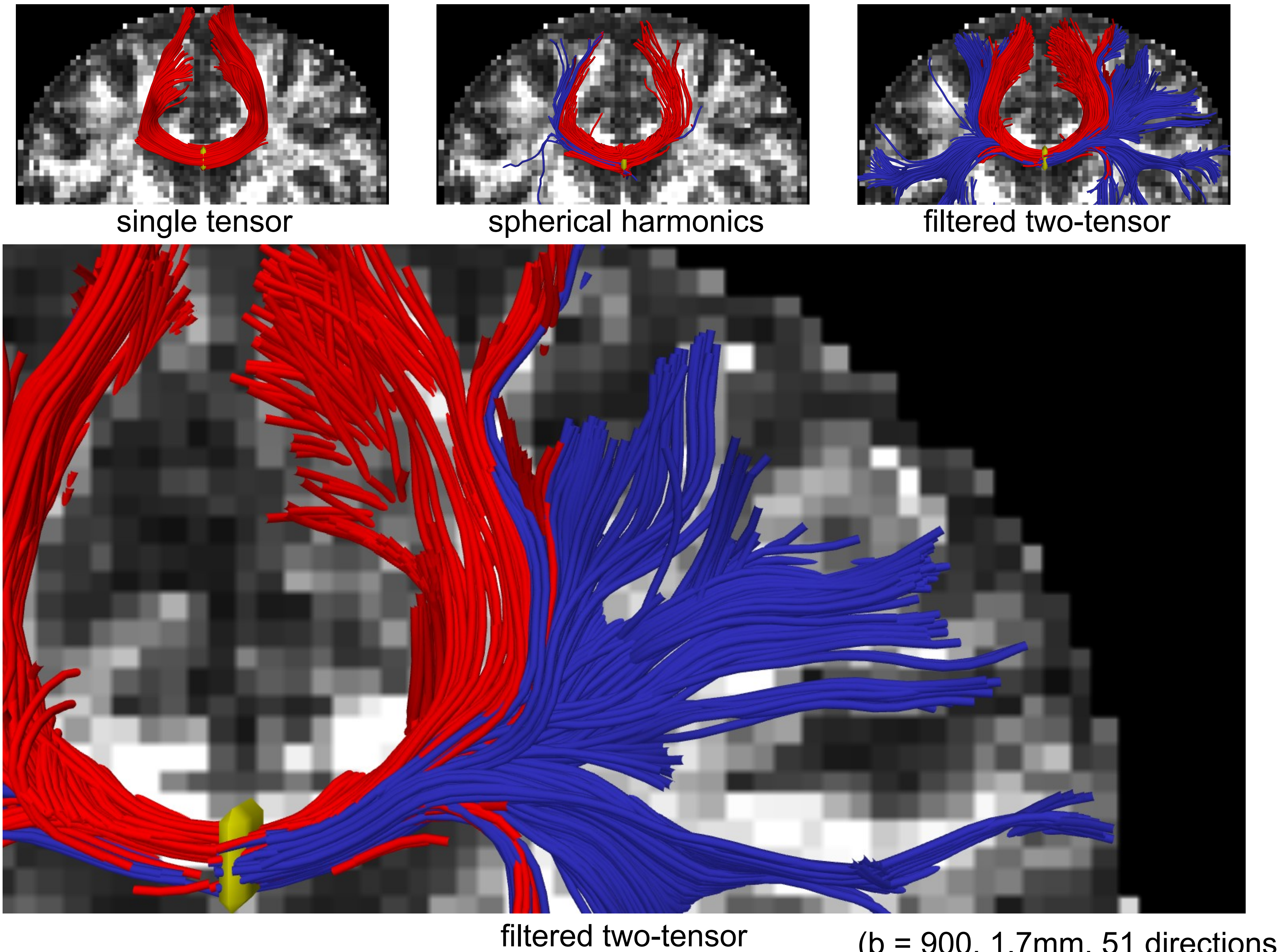
algorithm

interpolate scanner signal (measurement)

estimate model parameters with UKF

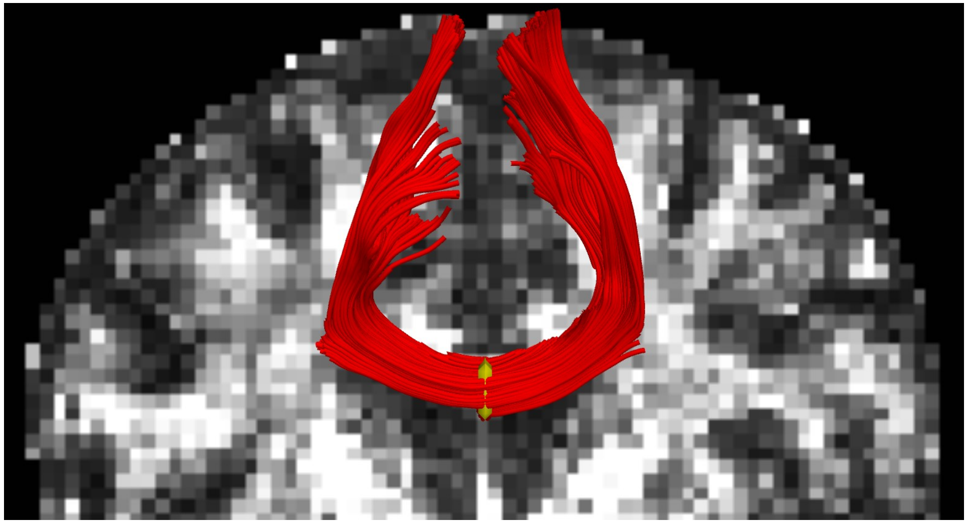
proceed in most consistent direction

terminate: FA < 0.15

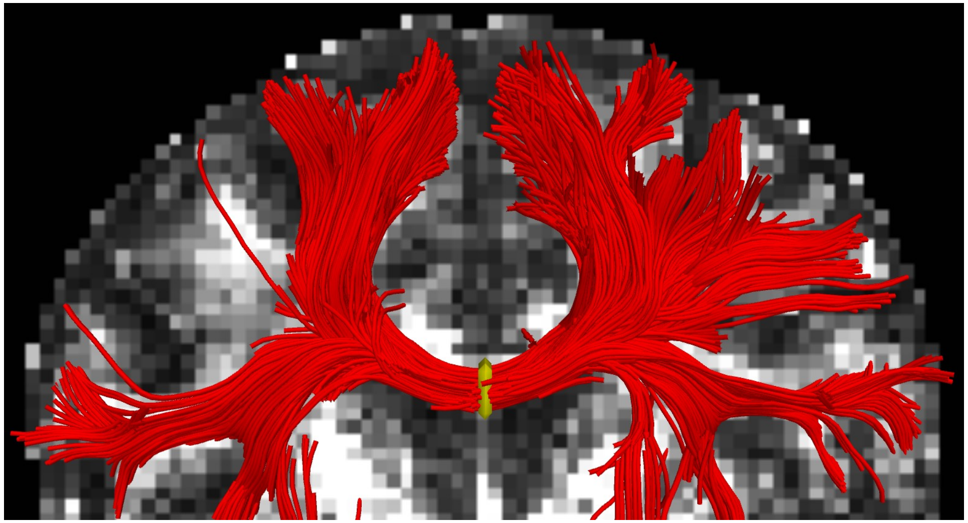

single tensor
spherical harmonics
filtered two-tensor
filtered two-tensor
(b = 900, 1.7mm, 51 directions)

fitting the local fiber model

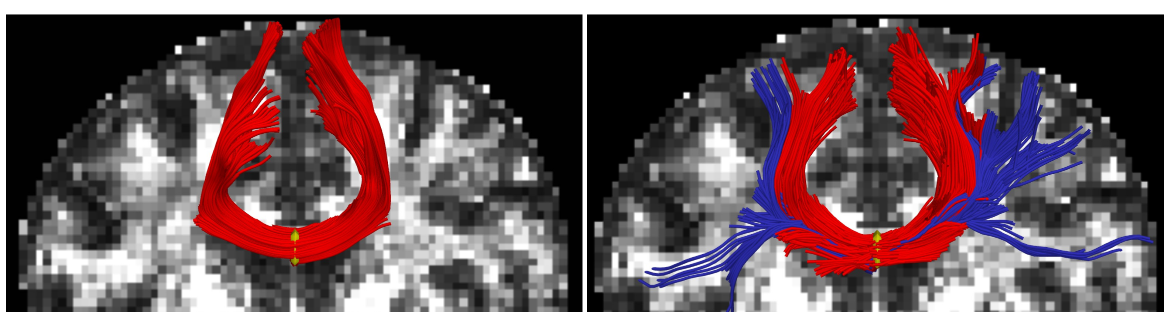
- Independent optimization v. filtered estimation
- Nonlinear model: Unscented Kalman filter



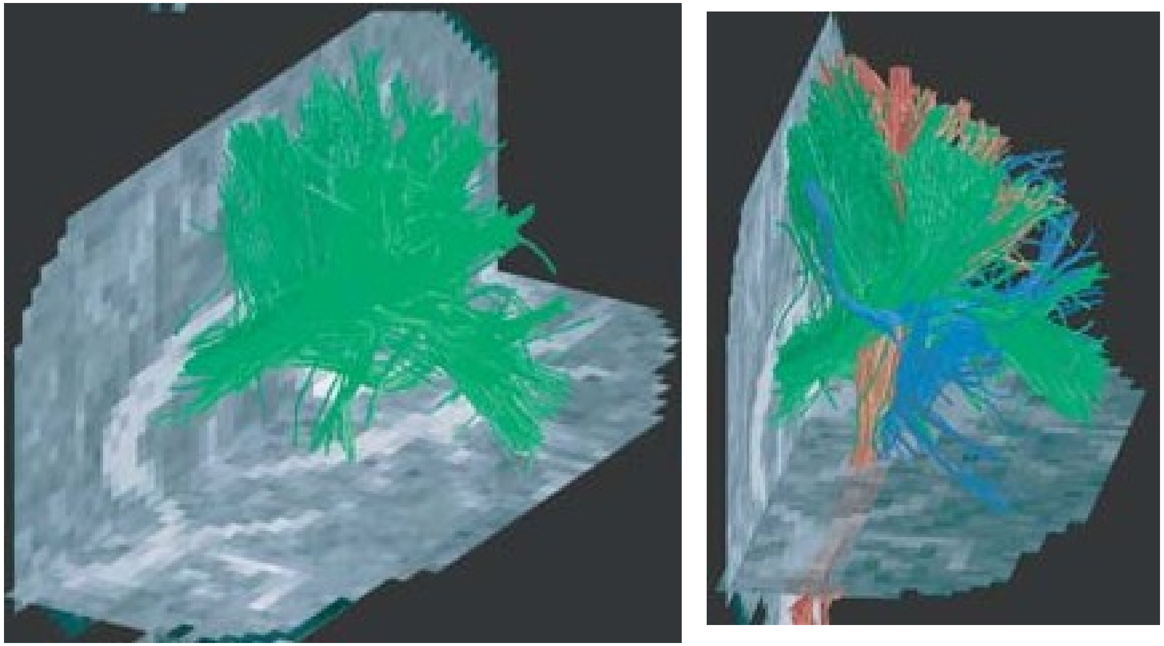
single-tensor



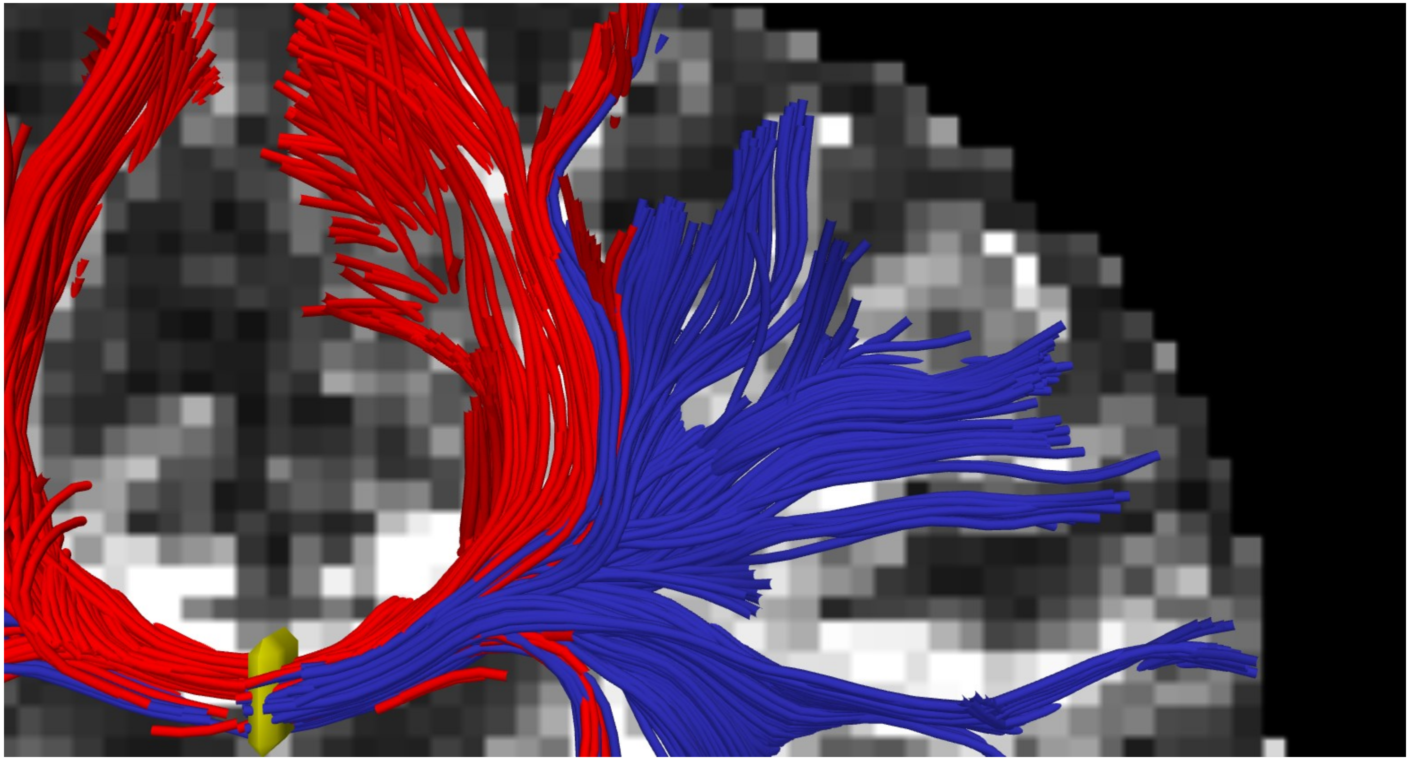
filtered two-tensor



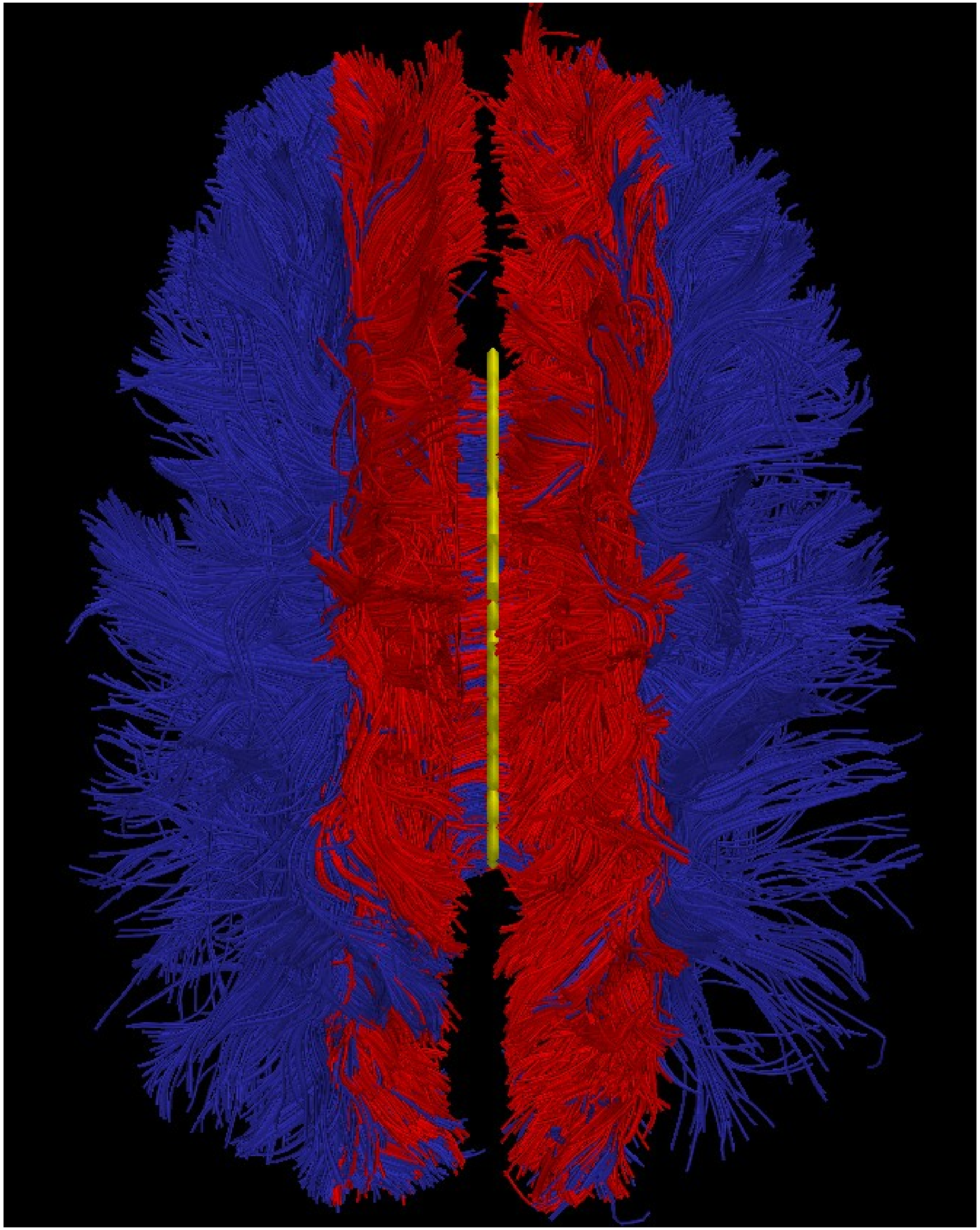
single tensor filtered two-tensor



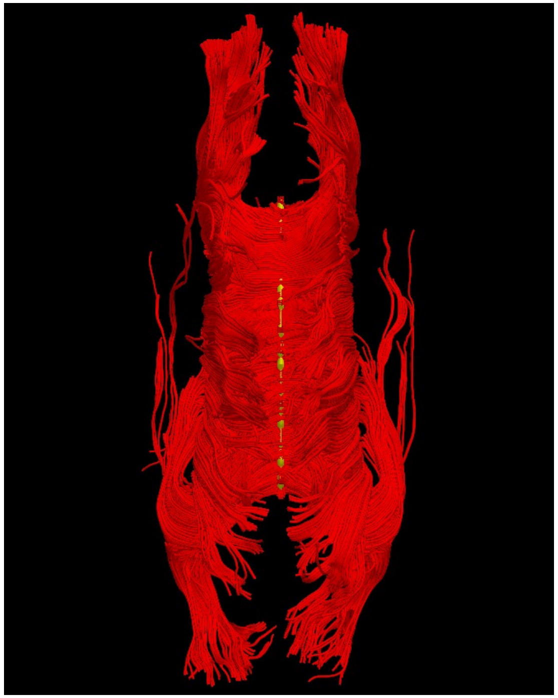
DSI [Hagmann 05]



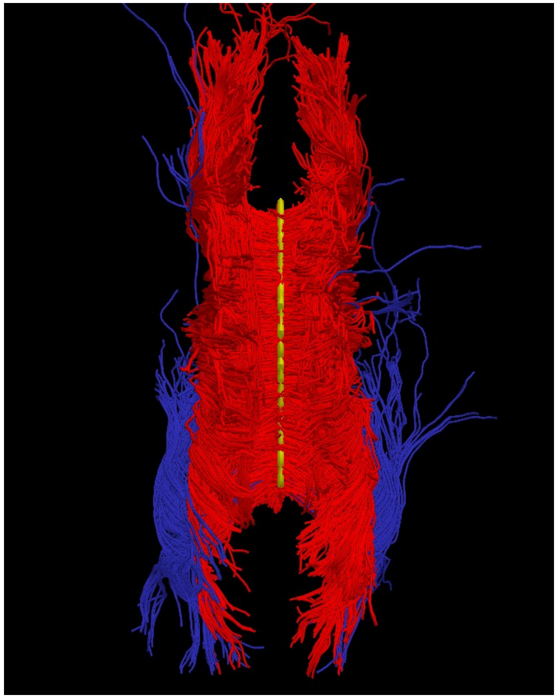
filtered two-tensor



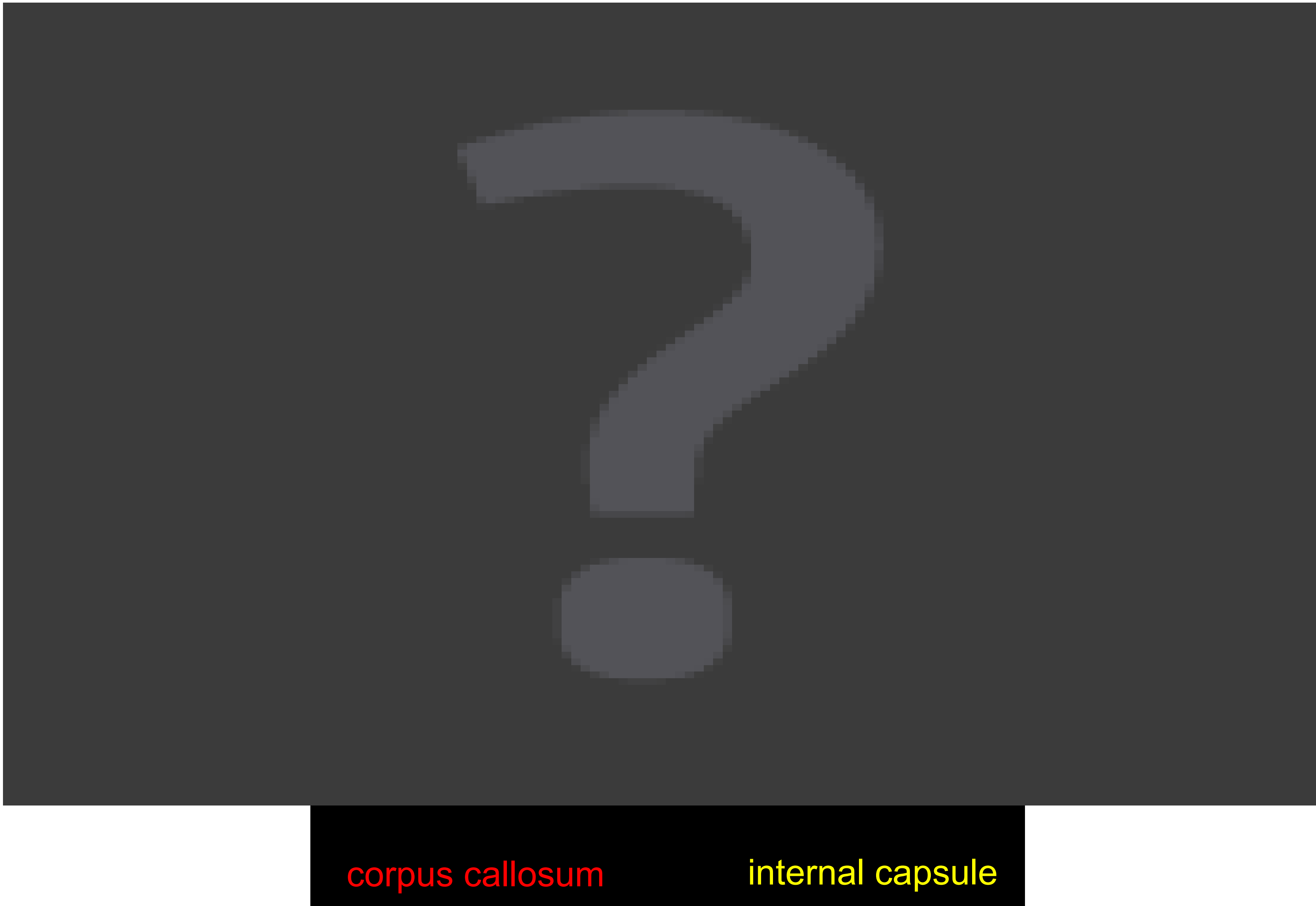
filtered two-tensor

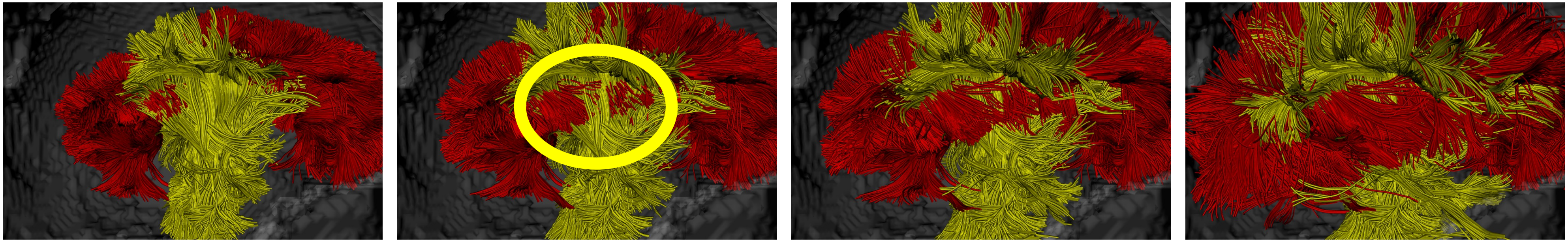


single tensor

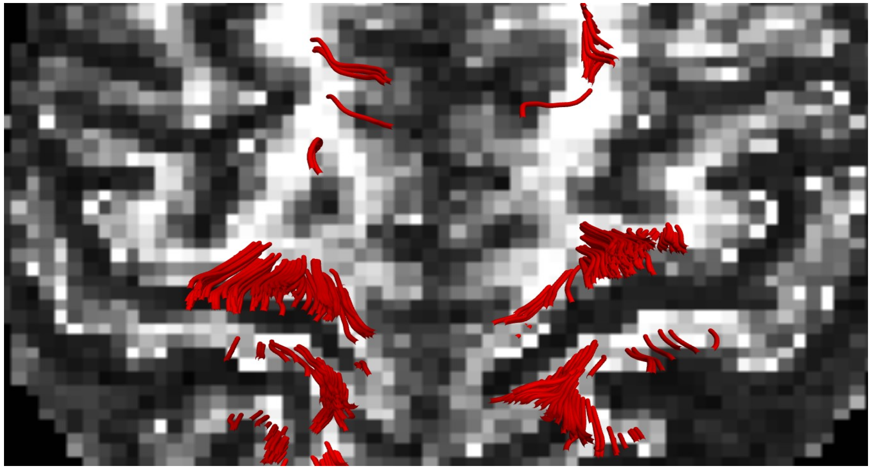


spherical harmonics

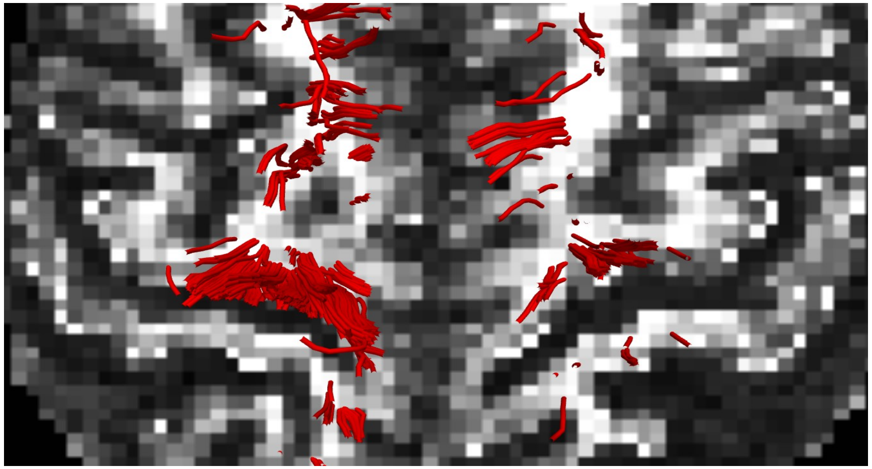

corpus callosum
internal capsule



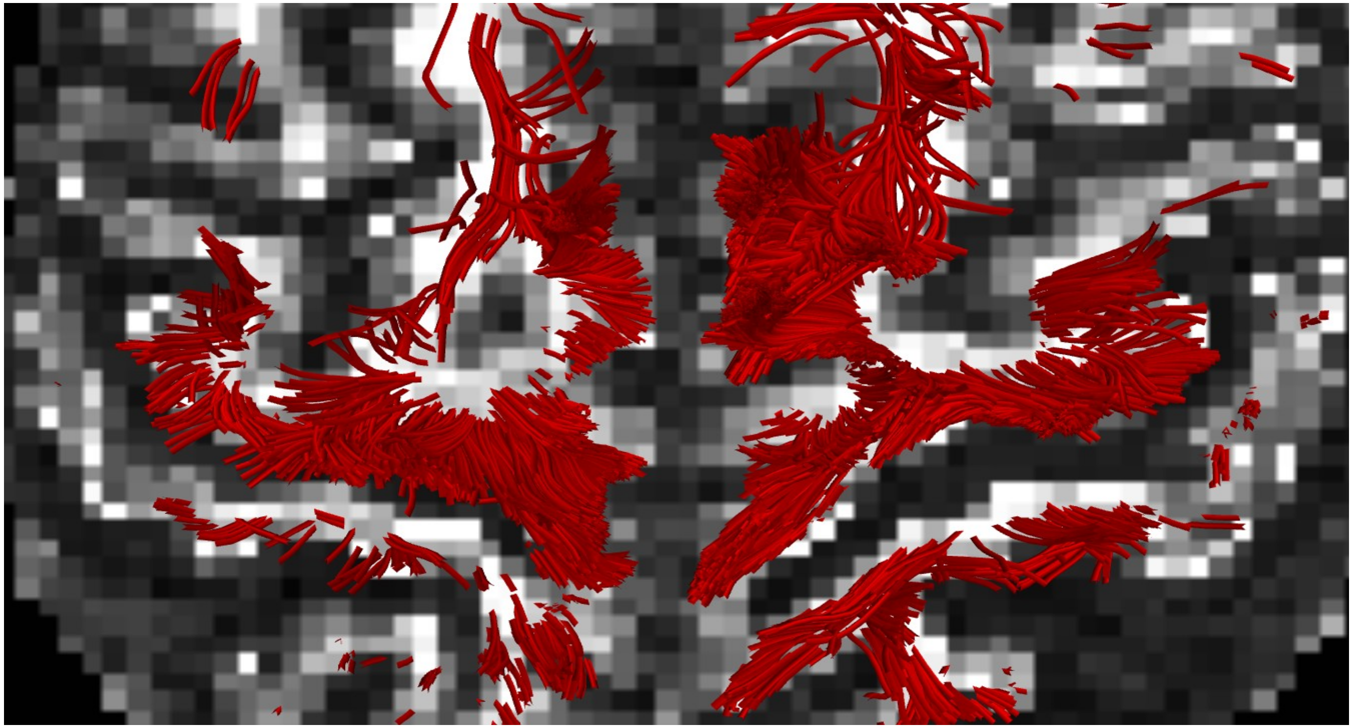
centrum semiovale



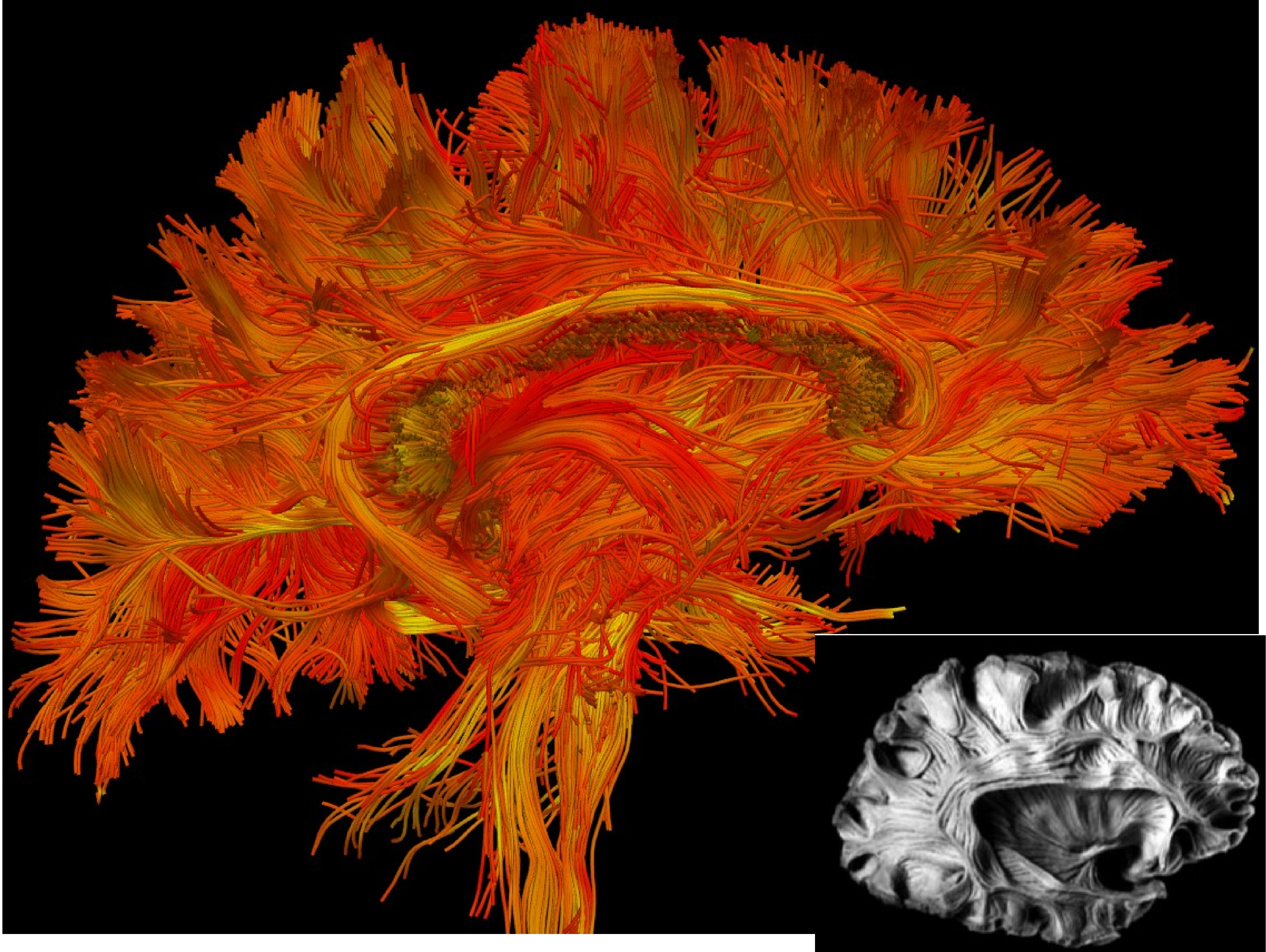
single tensor



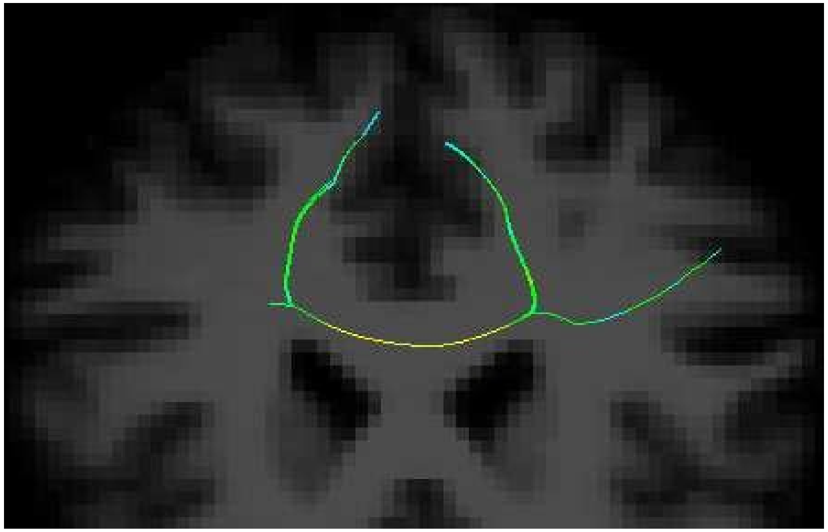
spherical harmonics



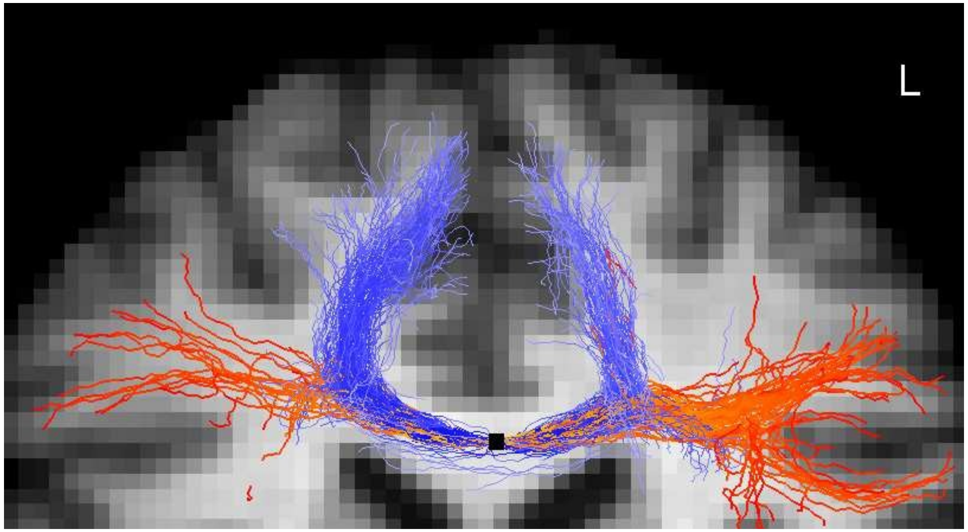
filtered two-tensor



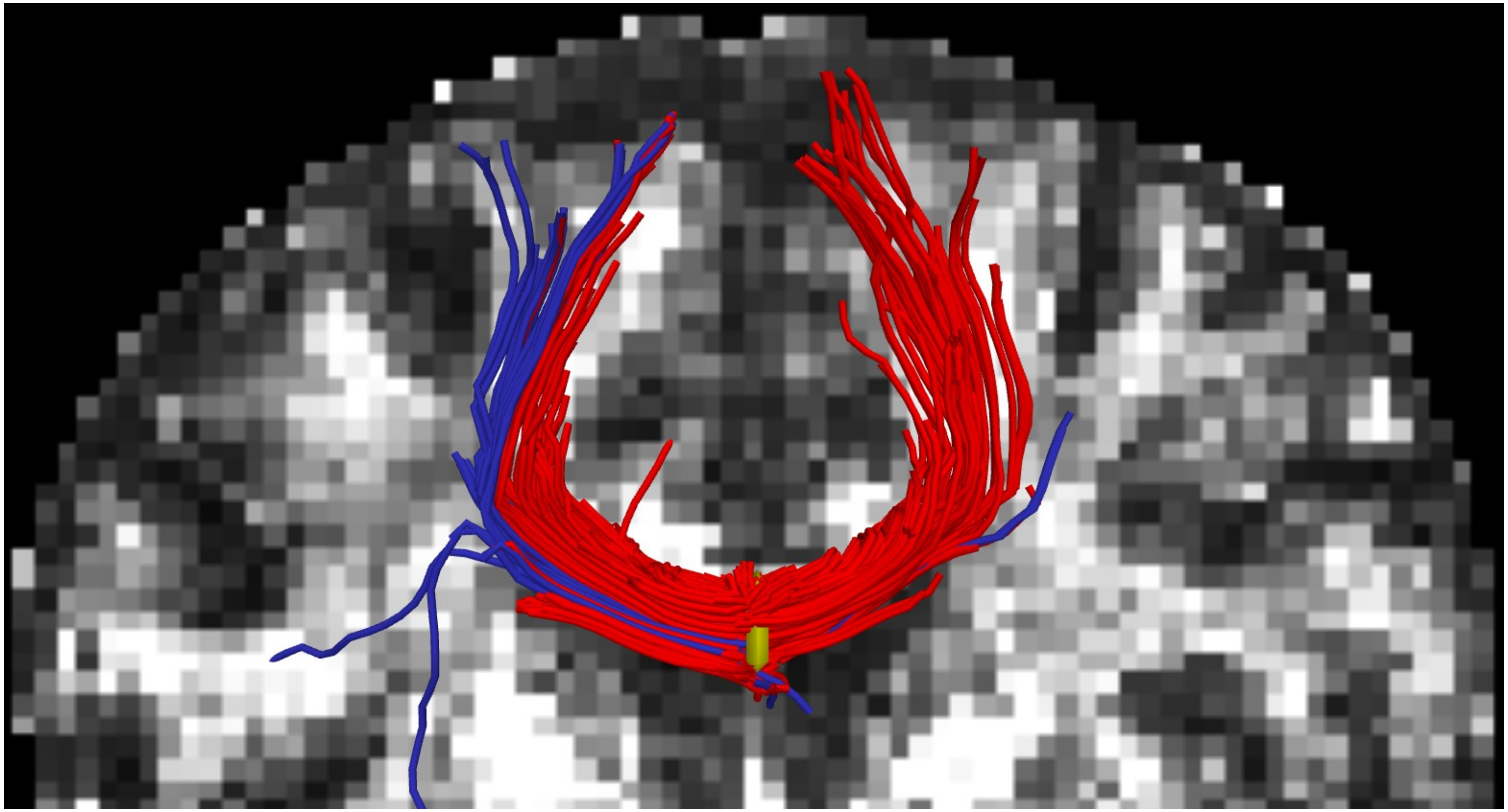
related work



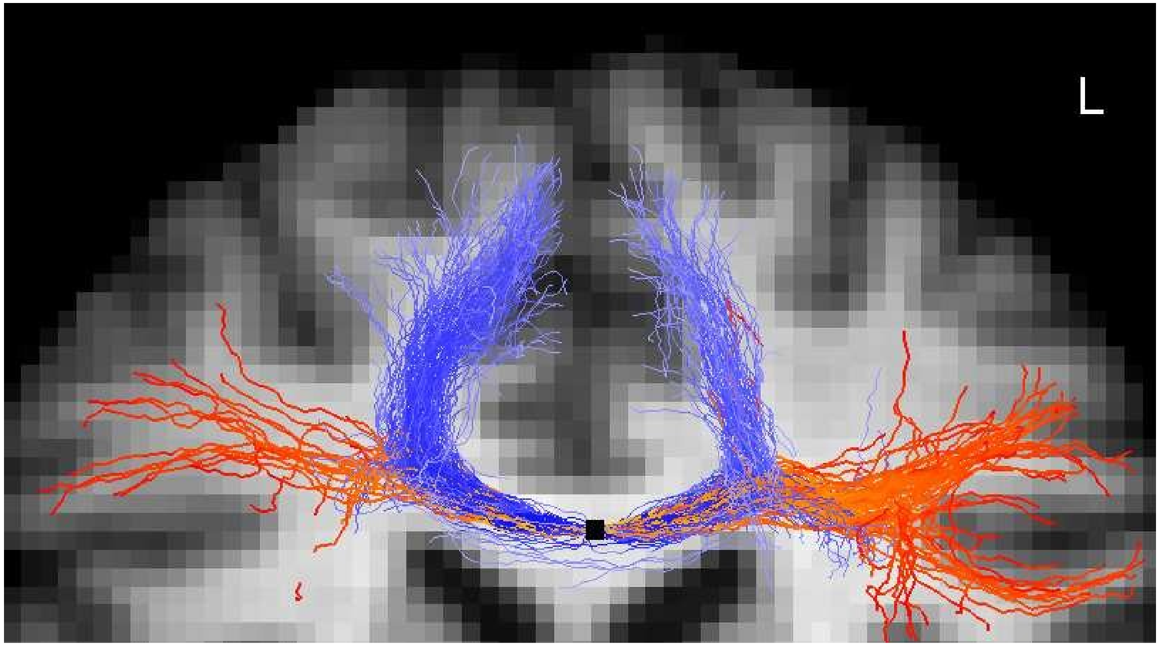
Deterministic (Descoteaux 2007)



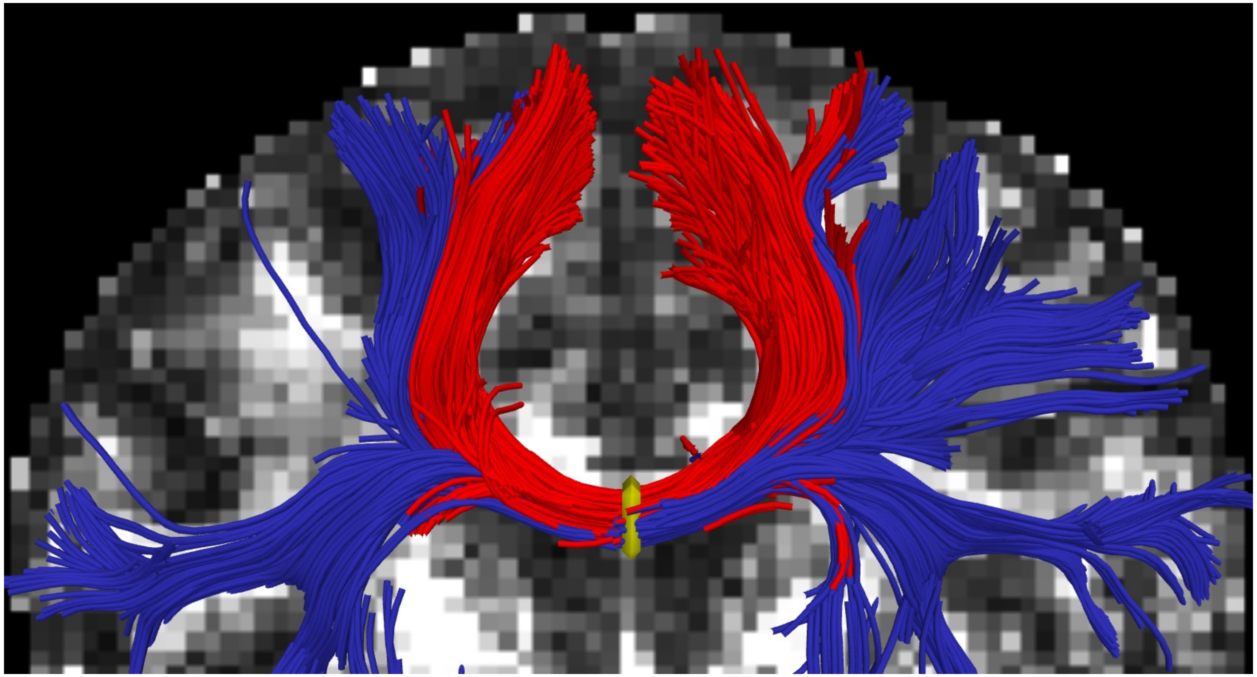
Probabilistic (Descoteaux 2007)



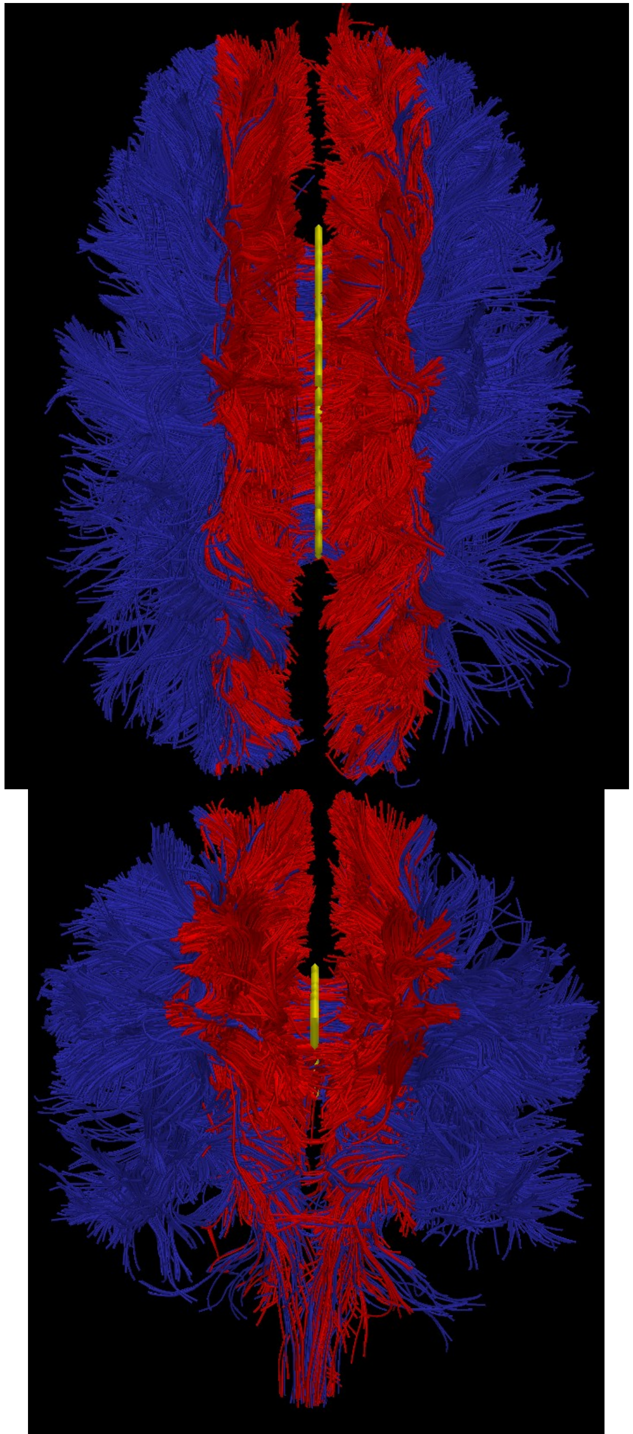
our implementation of spherical harmonics



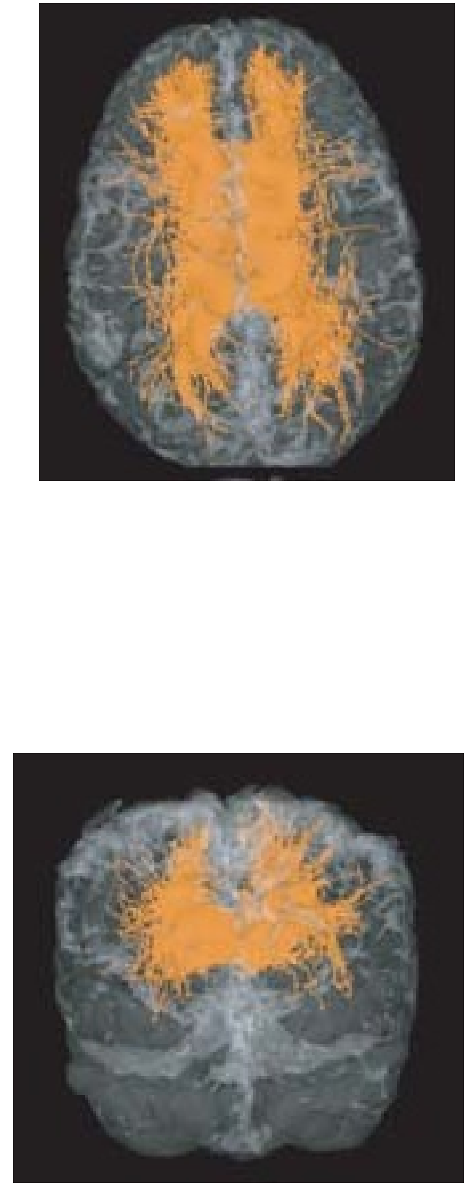
spherical harmonics, **probabilistic** tractography
[Descoteaux 2007]



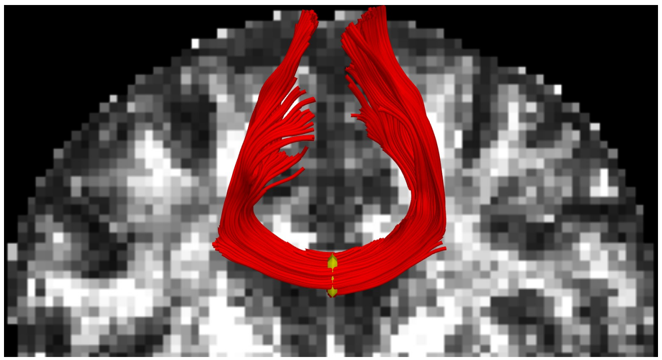
filtered two-tensor, **deterministic** tractography



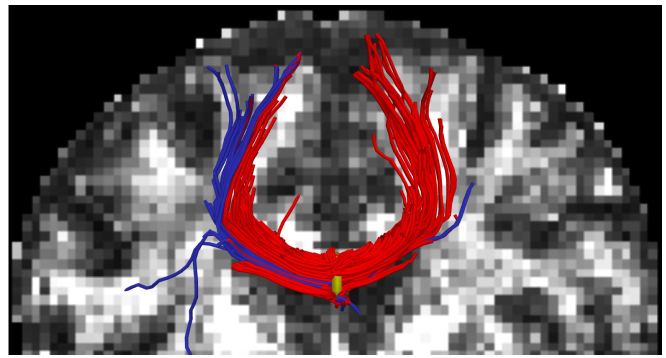
filtered two-tensor



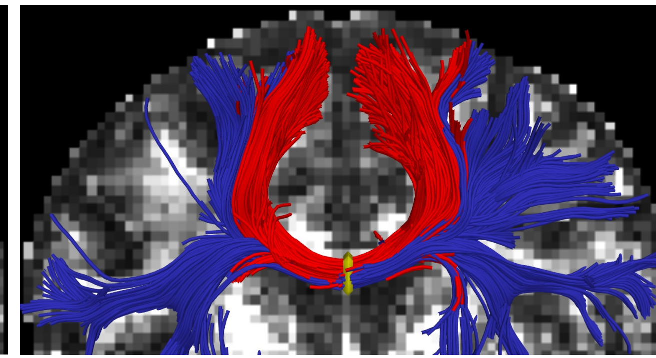
DSI [Hagmann 2005]



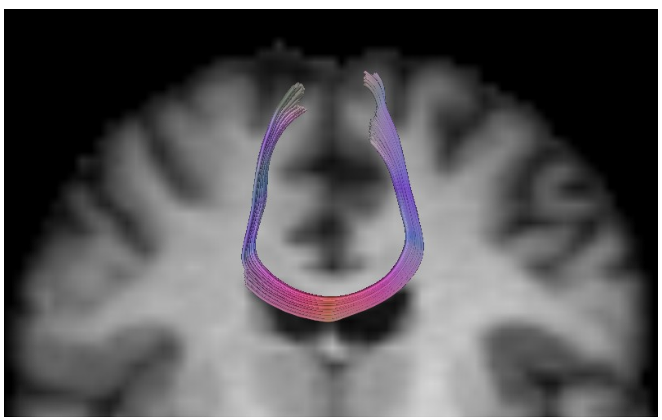
single tensor streamline



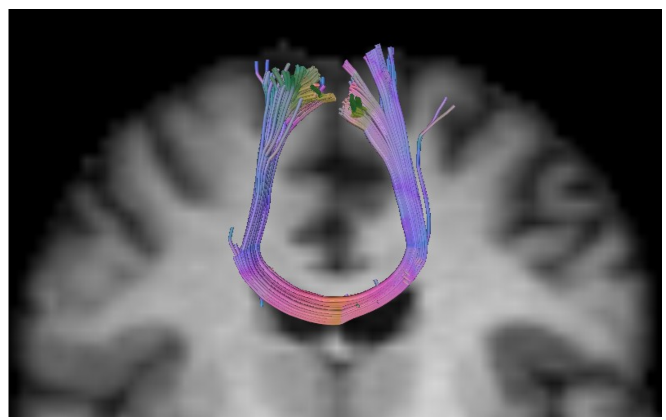
spherical harmonics



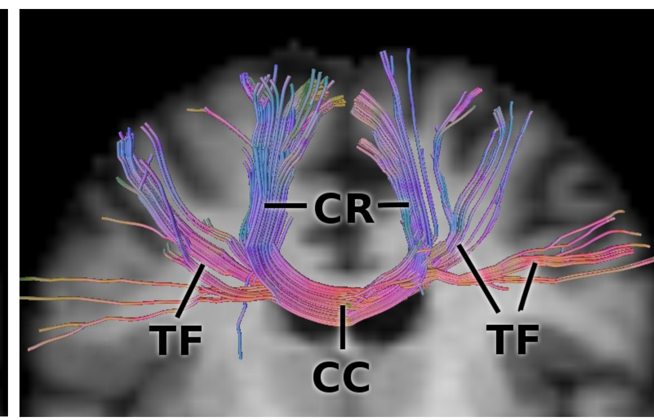
filtered two-tensor



single tensor streamline

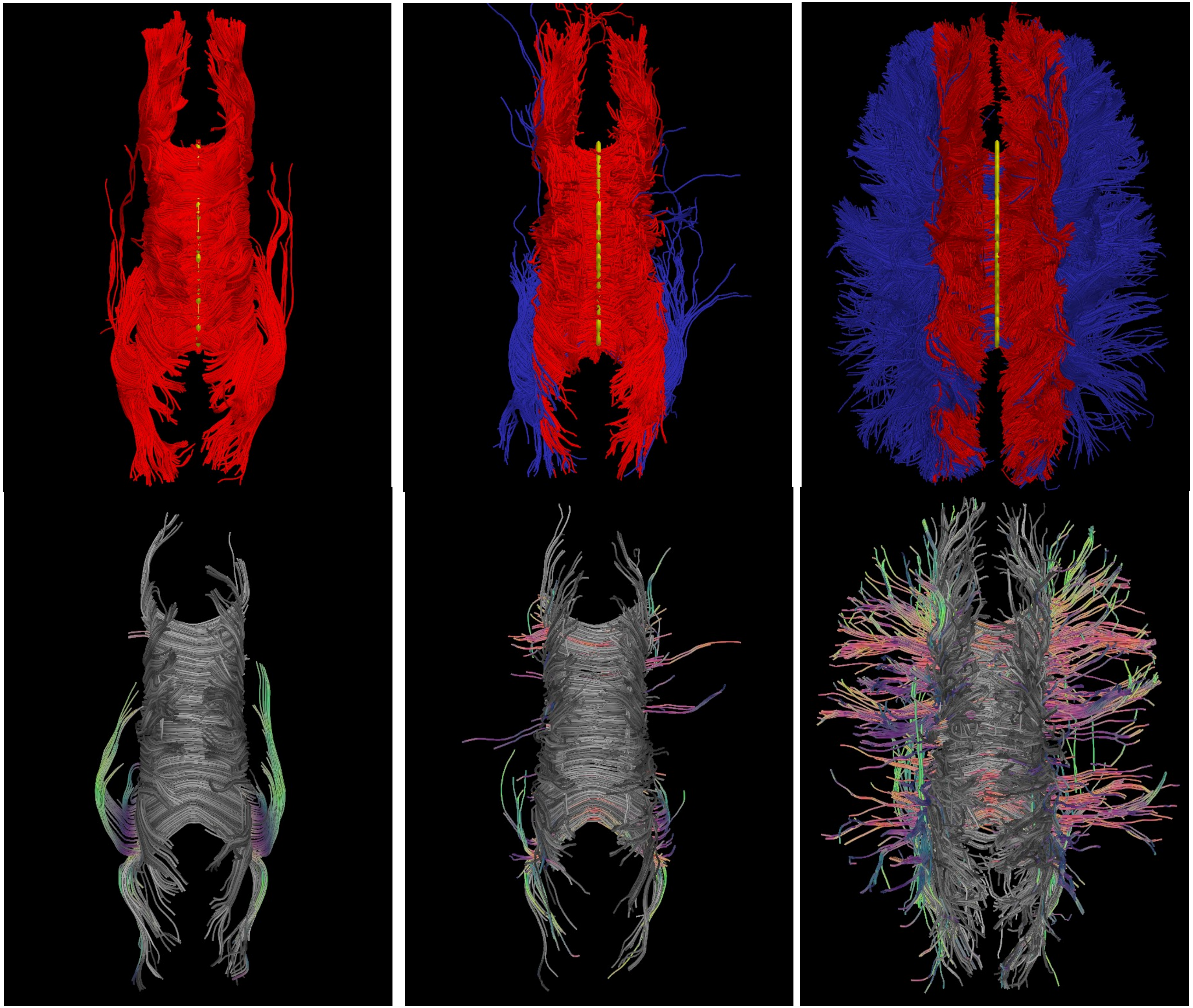


peak detection
(our "spherical harmonics")



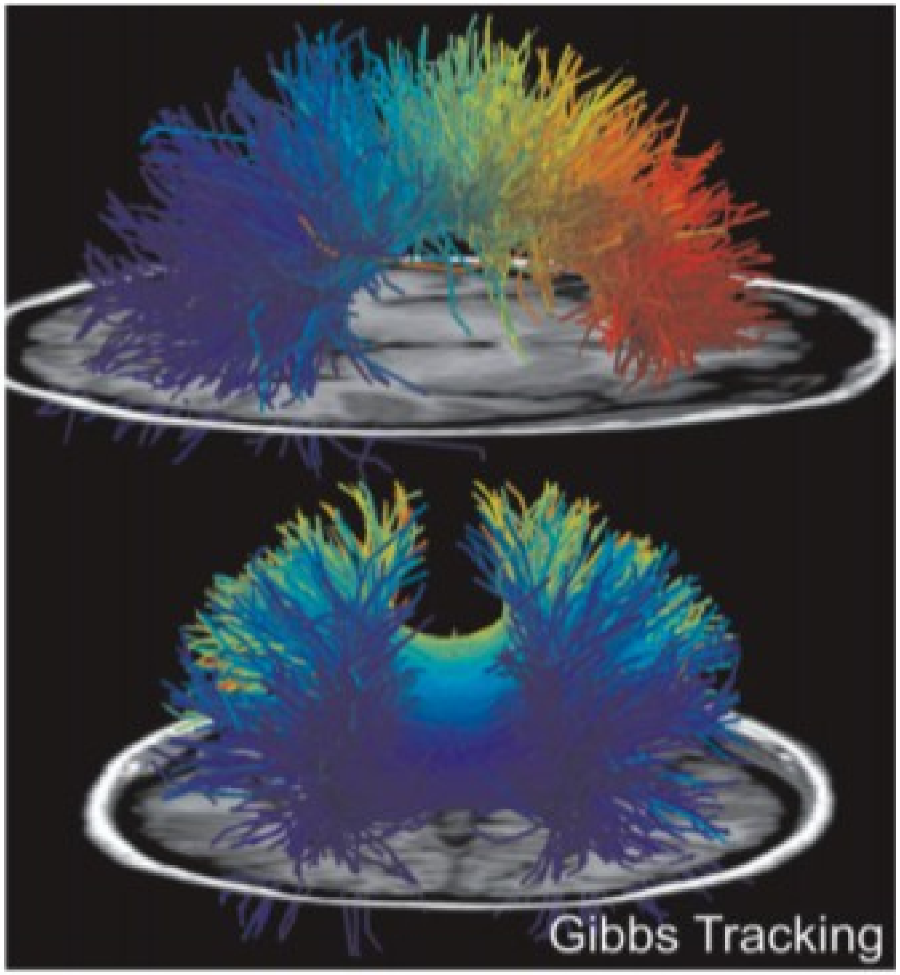
tensor decomposition

[Schultz 2008]



single tensor spherical harmonics tensor decomposition

[Schultz 2008]



[Kreher 08]

~1 month to compute

conclusion

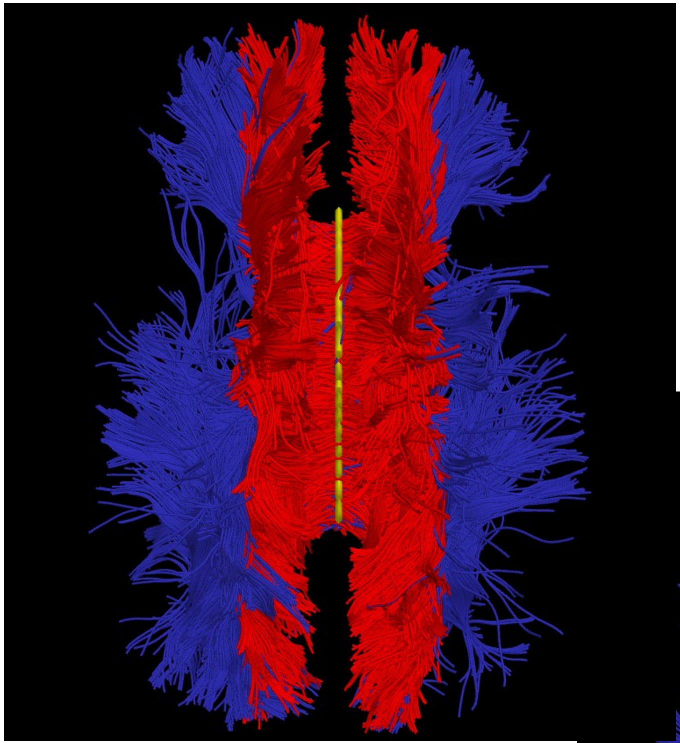
inherent coherence along the fiber

we should exploit it in the estimation

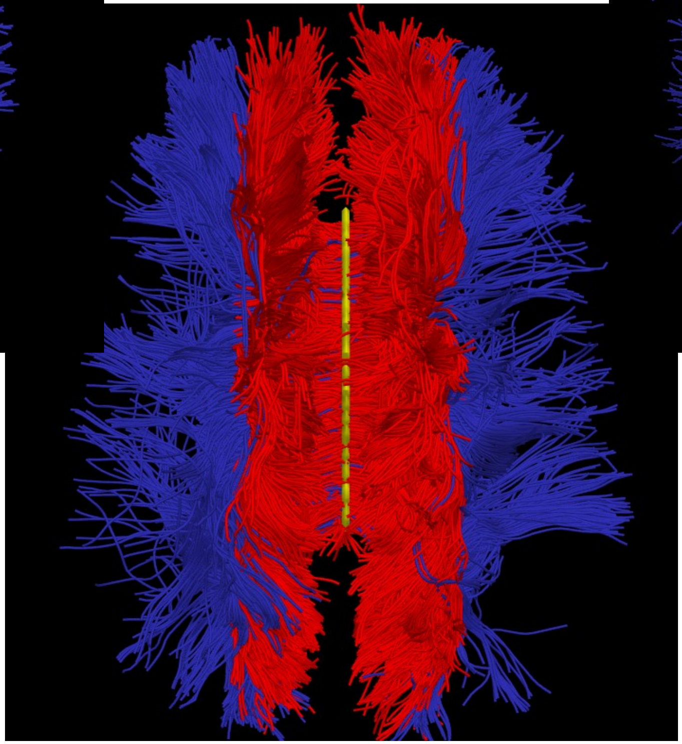
from the proposal...

- tensors: IPMI 2009, TMI 2010
- weighted mixtures: MICCAI 2009
- validation (phantom): DMFC (MICCAI) 2010
- Slicer3 module: Slicer 3.6 (?)
- population study: DMFC (MICCAI) 2010
- local voxel model, global path model: MICCAI 2010

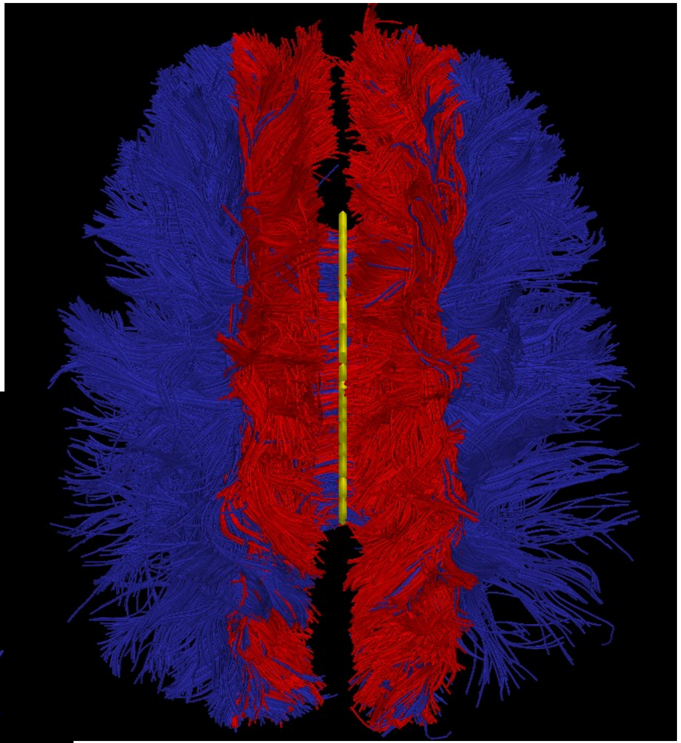
variations



Watson functions



weighted models



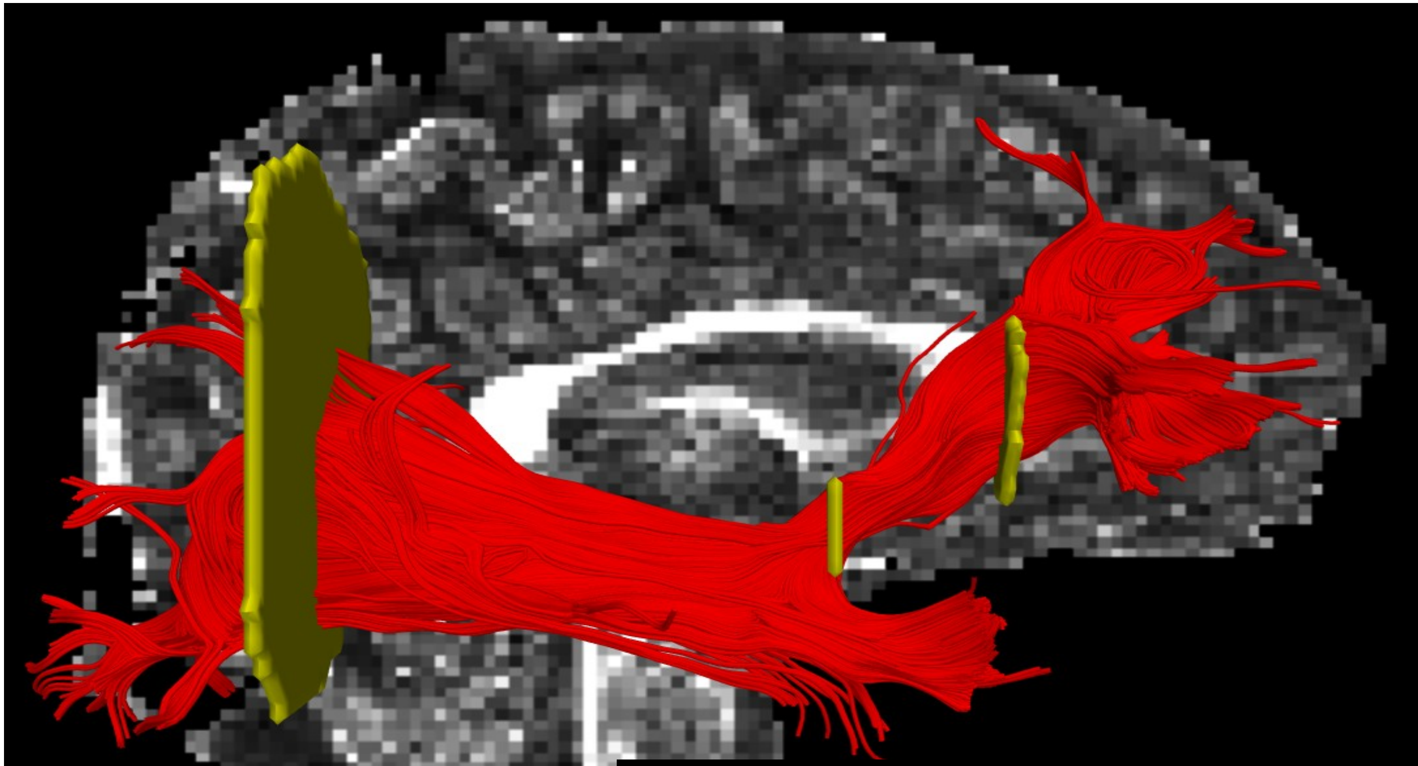
tensors

parametric:
- mixture models

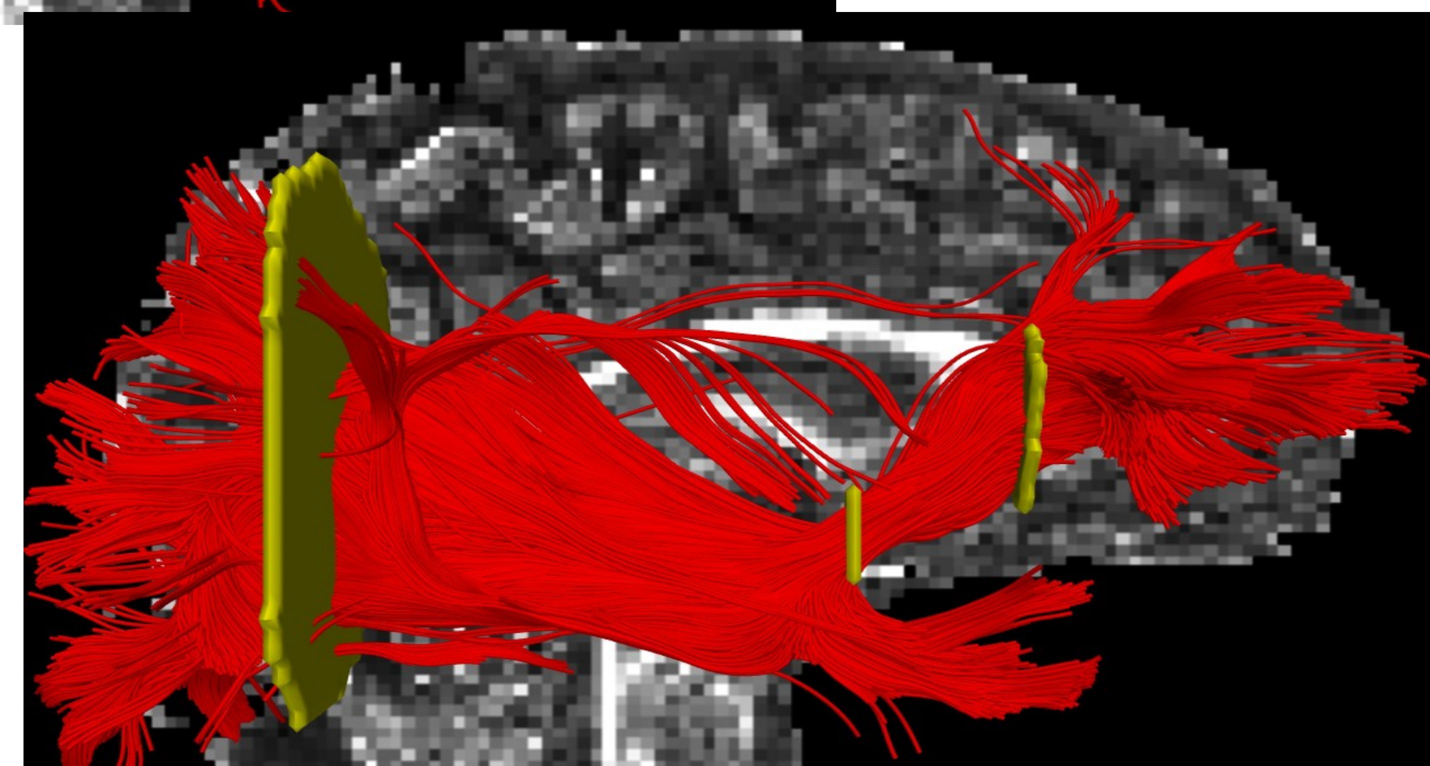
non-parametric:
- spherical harmonics
- higher-order

tensors

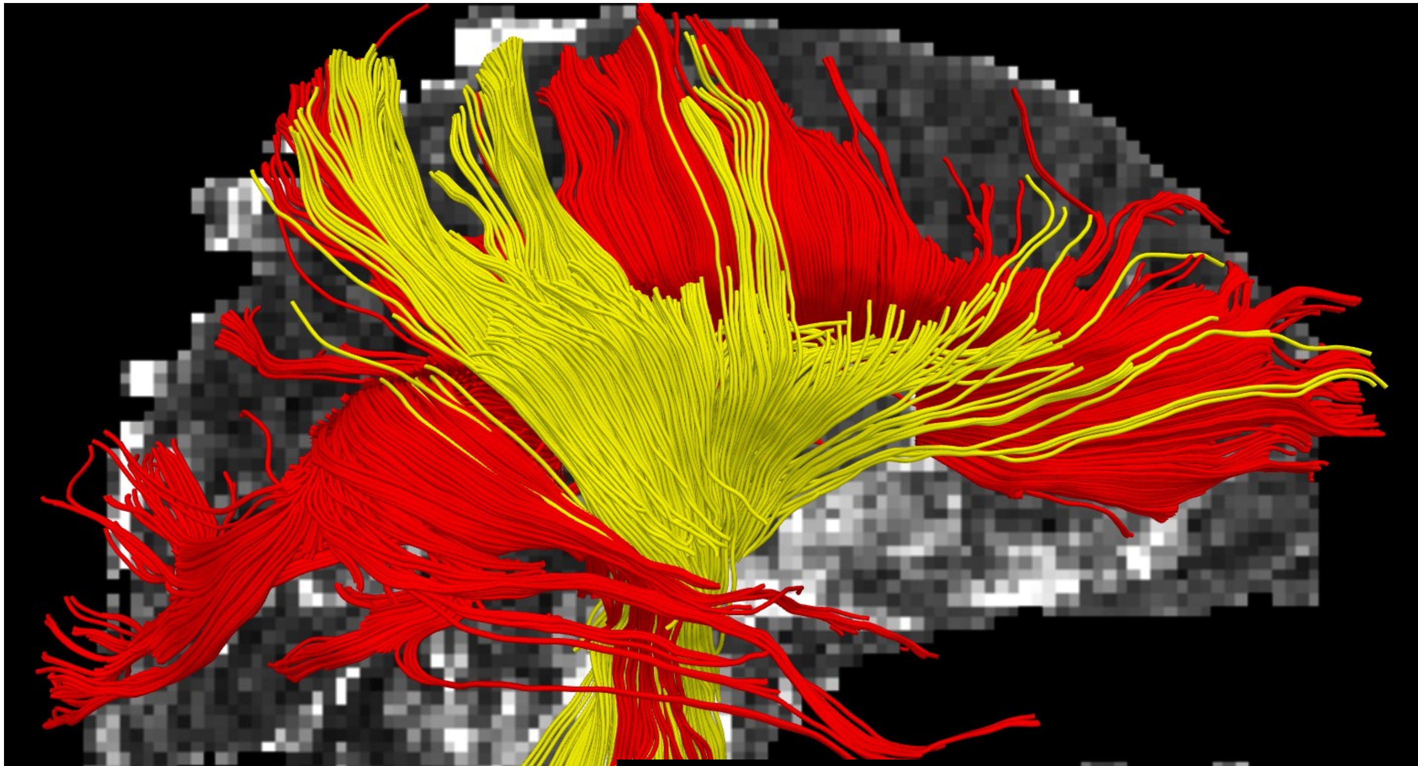
Malcolm, Shenton, Rathi. Two-tensor tractography using a constrained filter. In MICCAI, p.894–902, 2009.
Rathi, Malcolm, Michailovich, Westin, Shenton,Bouix. Tensor-kernels for simultaneous fiber model estimation and tractography. *Magnetic Resonance in Medicine*, 64(1):138–148, 2010.



streamline
single-tensor

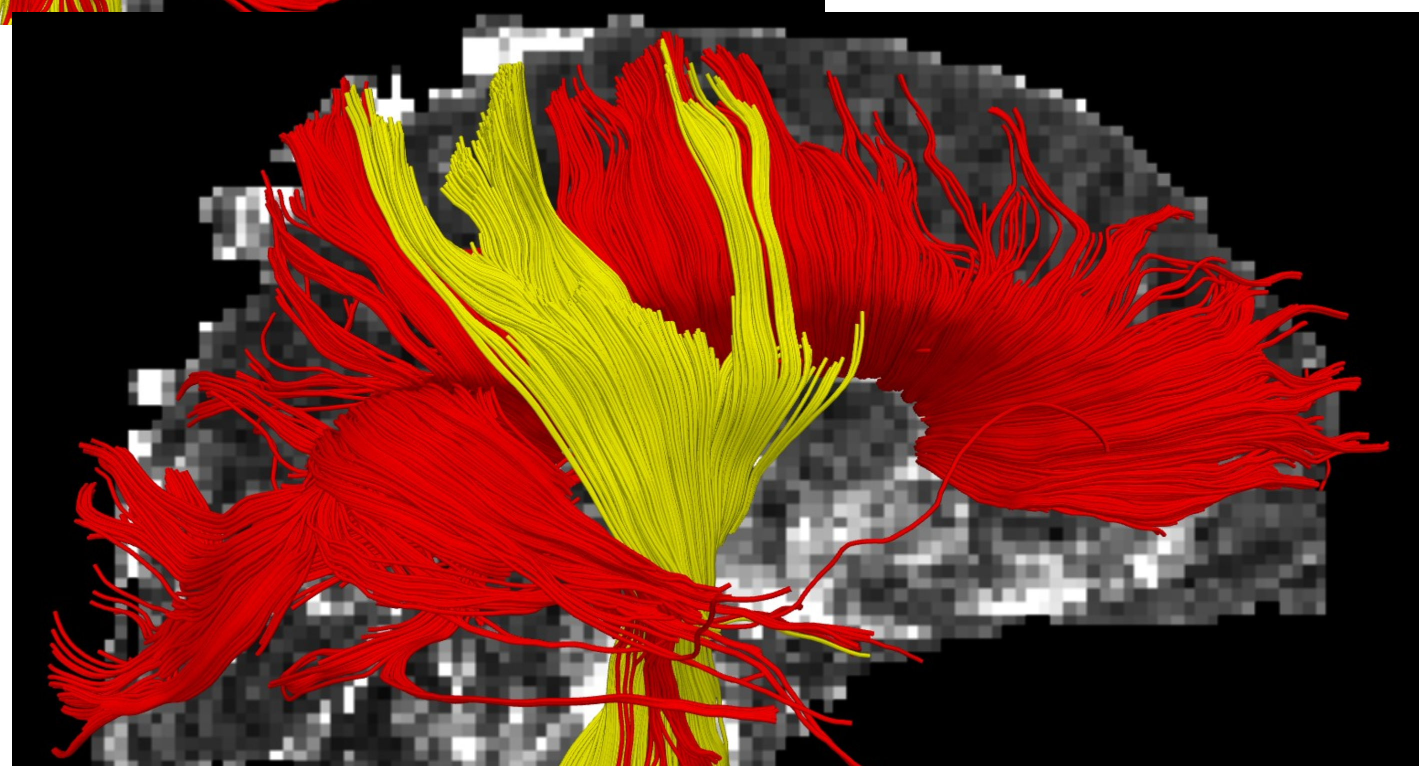


filtered
single-tensor



streamline
single-tensor

filtered
single-tensor



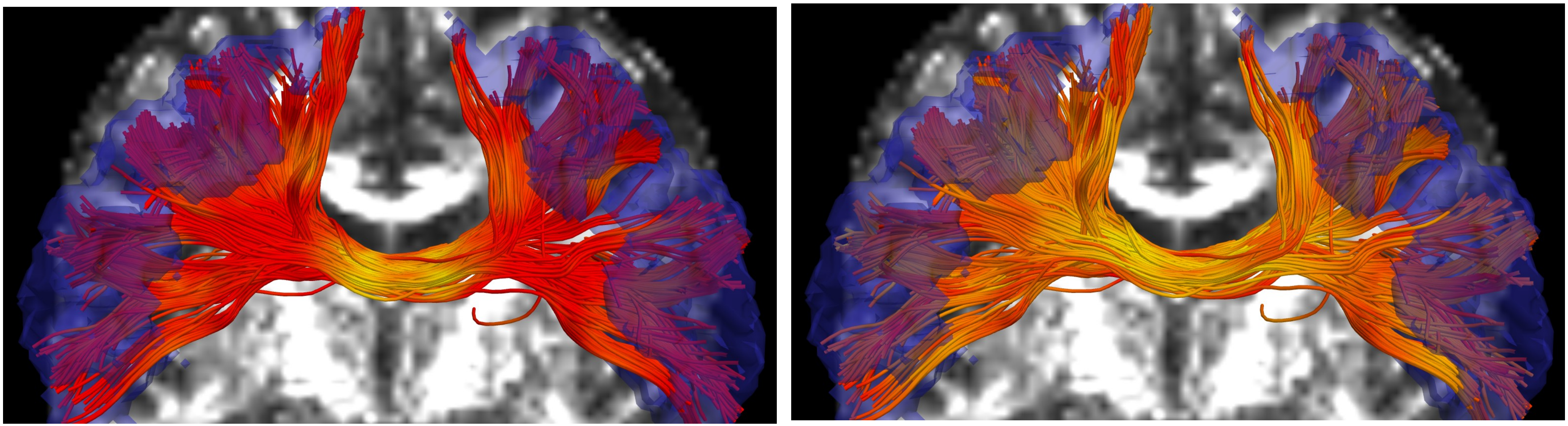
model components
one or two?

a population study

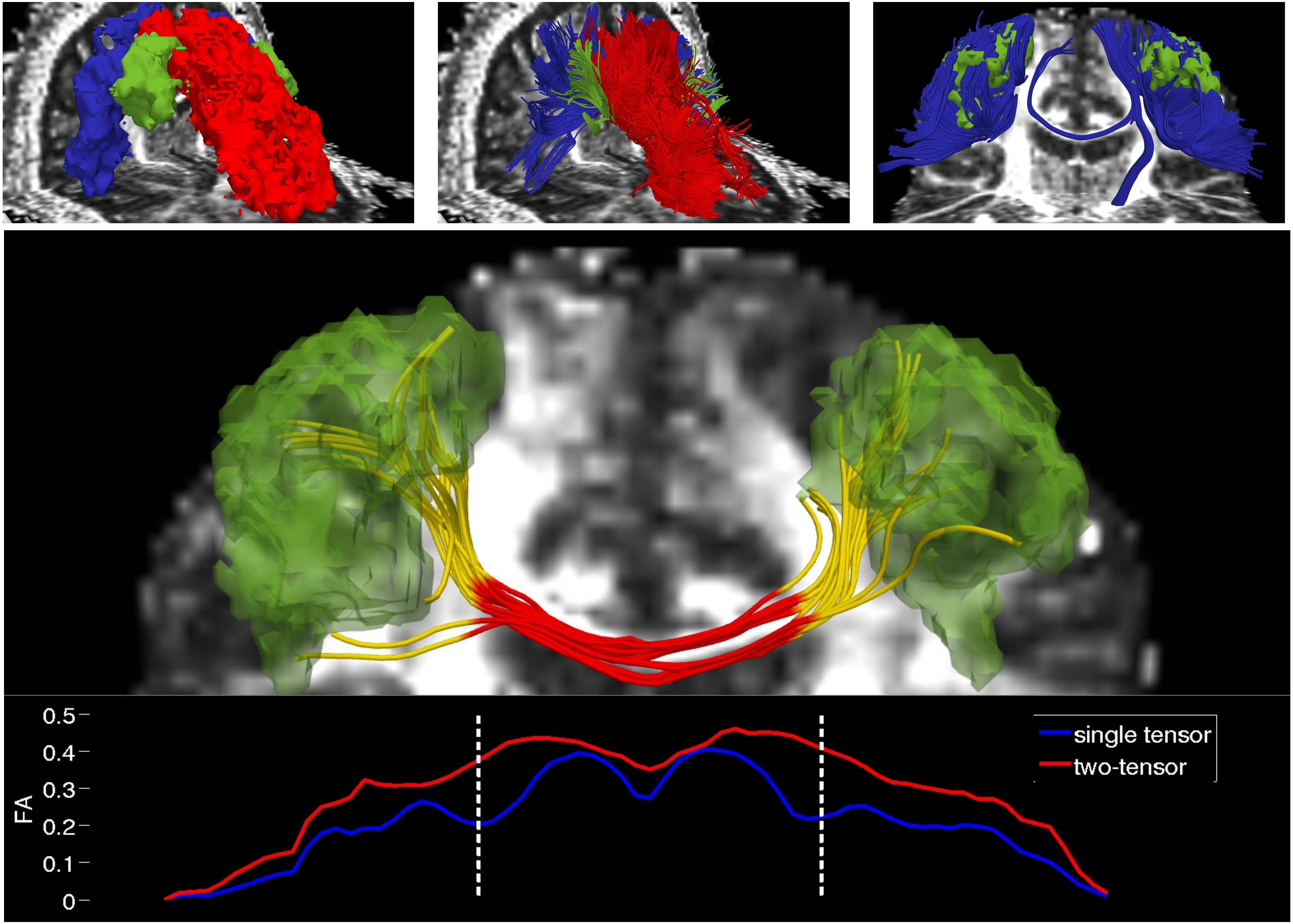
Malcolm, Kubicki, Shenton, Rathi. The effect of local fiber model on population studies. In Diffusion Modeling and Fiber Cup (in MICCAI), pages 33–40, 2009.

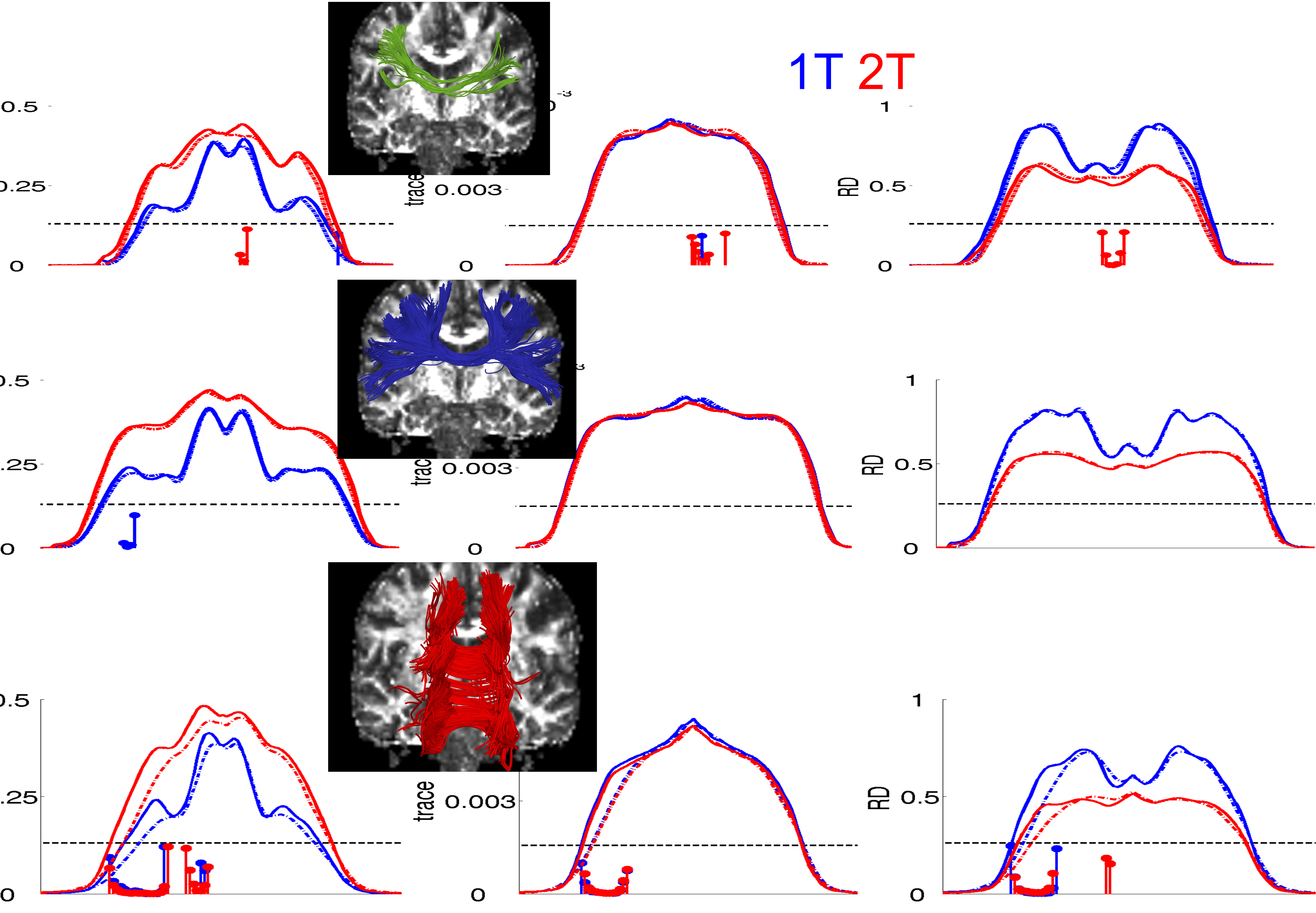
Rathi, Malcolm, Michailovich, Goldstein, Seidman, McCarley, Westin, Shenton. Biomarkers for identifying first-episode schizophrenia patients using diffusion weighted imaging. In MICCAI, volume 6361, pages 657–665, 2010.

Experiment: look at FA superimposed on two-tensor fibers

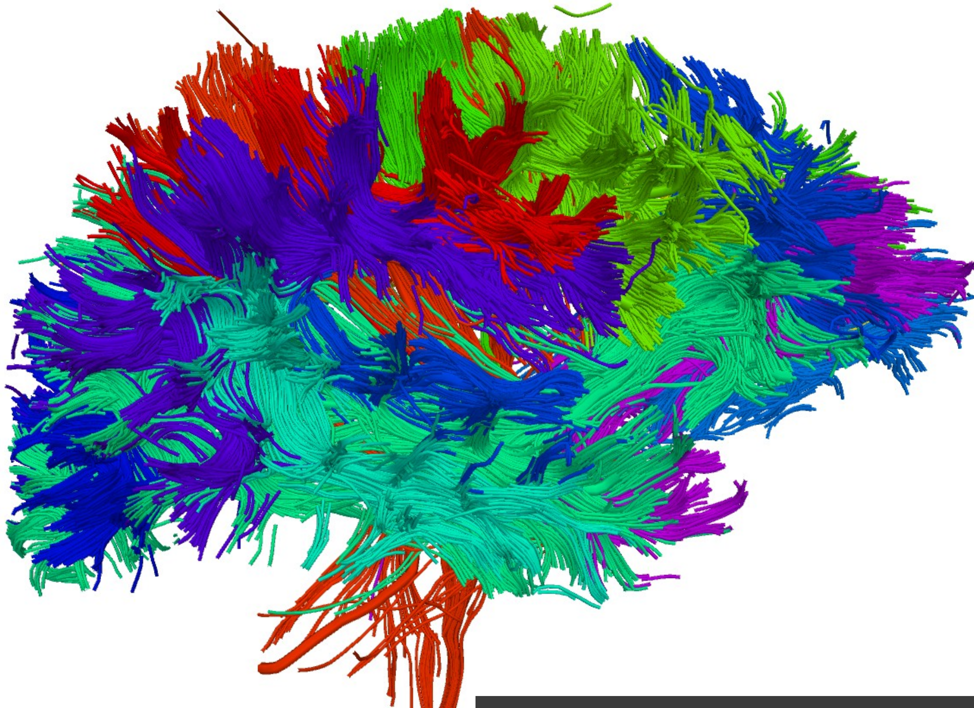


Result: single-tensor FA drops outside corpus callosum while two-tensor maintains higher FA on out to gray-matter


0.5
0.4
0.3
0.2
0.1
0
FA
single tensor
two-tensor



1T 2T
0.5
0.25
0
trace
0.003
0
1
RD
0.5
0



J. G. Malcolm, M. E. Shenton, and Y. Rathi. Filtered tractography: State estimation in a constrained subspace. In Diffusion Modeling and Fiber Cup (in MICCAI), pages 122–133, 2009.

end

extra...

the watson approximation

J. G. Malcolm, O. Michailovich, S. Bouix, C.-F. Westin, M. E. Shenton, and Y. Rathi. A filtered approach to neural tractography using the Watson directional function. *Medical Image Analysis*, 14:58–69, 2010.

do we need six parameters?

- for tractography, care most about direction
  - 3 parameters
- approximate diffusivity with scalar anisotropy
  - fractional anisotropy
  - 1 parameter

Watson approximation

$$S(\boldsymbol{u}) = s_0 e^{-b\boldsymbol{u}^T D\boldsymbol{u}}$$

$$\begin{aligned} -b\boldsymbol{u}^T D\boldsymbol{u} &\approx -b\lambda_1 \boldsymbol{u}^T(\boldsymbol{m}\boldsymbol{m}^T)\boldsymbol{u} \\ &= -b\lambda_1(\boldsymbol{u}^T\boldsymbol{m})(\boldsymbol{m}^T\boldsymbol{u}) \\ &= -k(\boldsymbol{u}^T\boldsymbol{m})^2 \end{aligned}$$

$$S(\boldsymbol{u}) = e^{-k(\boldsymbol{u}^T\boldsymbol{m})^2}$$

principal direction $\boldsymbol{m} \in \mathbb{R}^3$

scaling factor $k \in \mathbb{R}$

4 parameters

[Rathi 09, Malcolm 10]

model parameters

for two fibers...

...two principal directions $\boldsymbol{m} \in \mathbb{R}^3$

...two scaling factors $k \in \mathbb{R}$

4 + 4 = 8 parameters

model parameters

for two fibers...

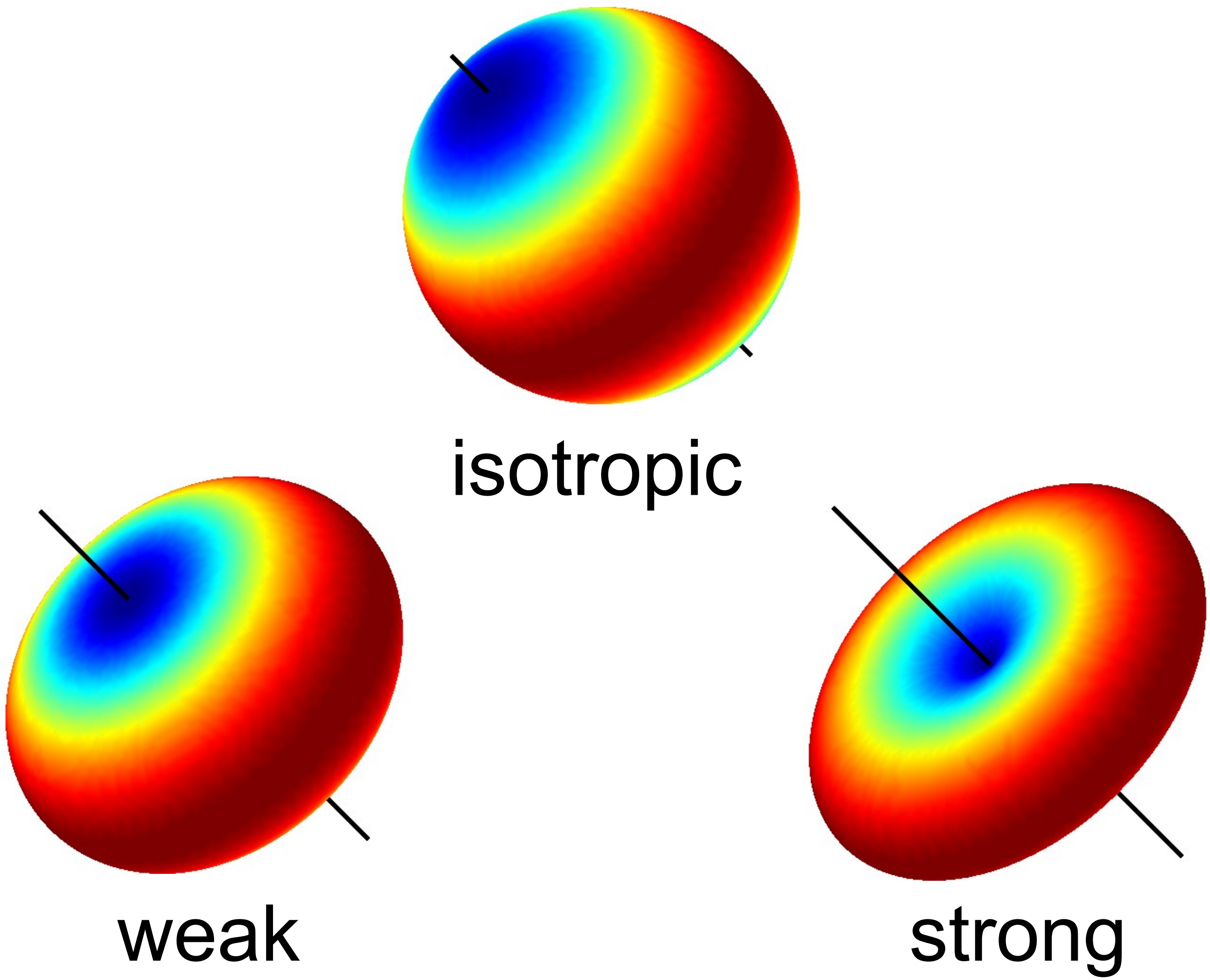
...two principal directions $\boldsymbol{m} \in \mathbb{R}^3$

...two scaling factors k $\in \mathbb{R}$

4 + 4 = 8 parameters

$$S(\boldsymbol{u}) = 0.5\, e^{-k_1 (\boldsymbol{u}^T \boldsymbol{m}_1)^2} + 0.5\, e^{-k_2 (\boldsymbol{u}^T \boldsymbol{m}_2)^2}$$

effect of scaling factor **k**



synthetic validation

brute force optimization

- matching pursuit
- parametric dictionary
- noiseless signal
- discretization, noise

spherical harmonics

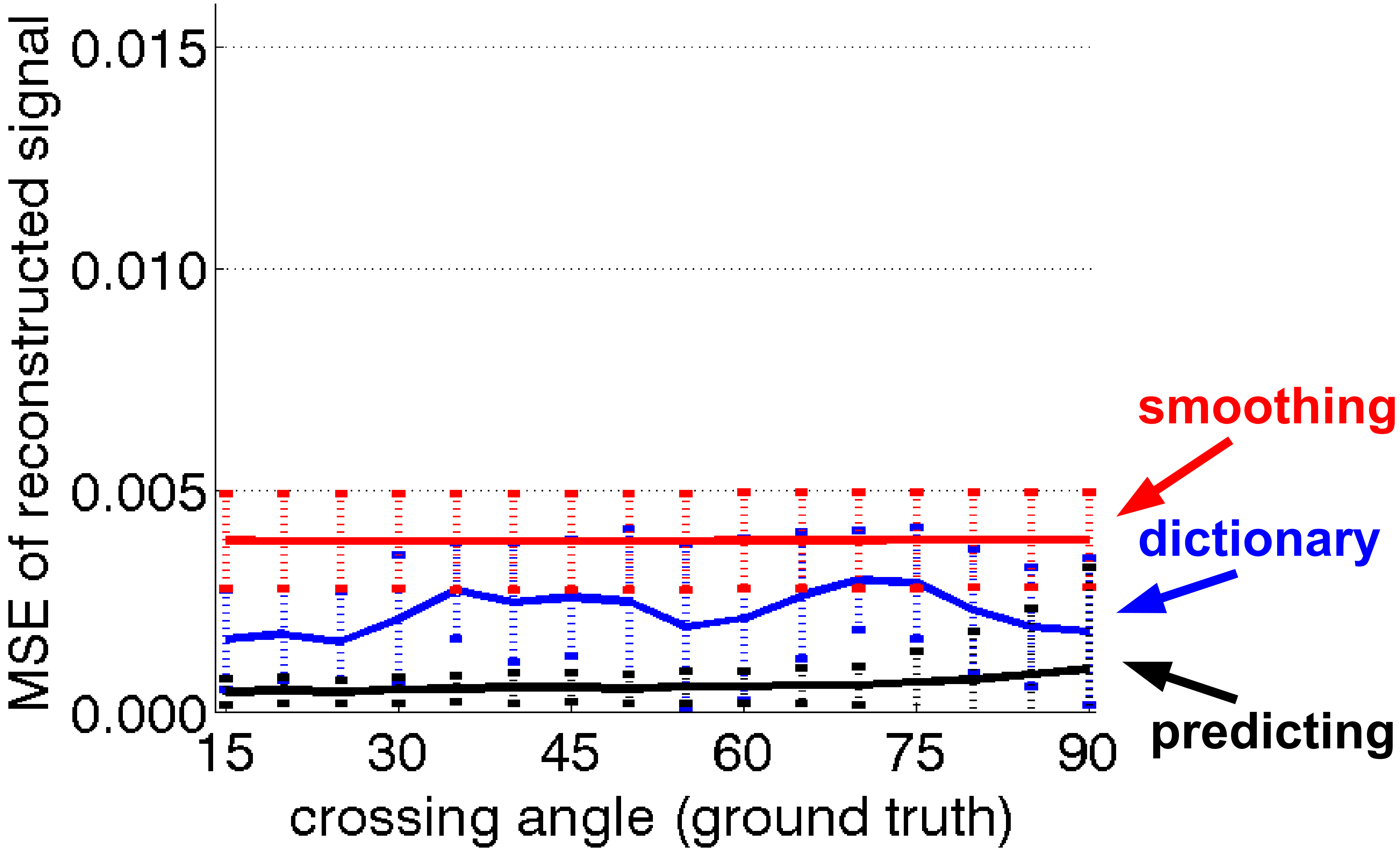
- non-parametric
- order eight (8)
- fiber sharpening for peak detection (L=0.006)

[Descoteaux 07]

filtered tractography

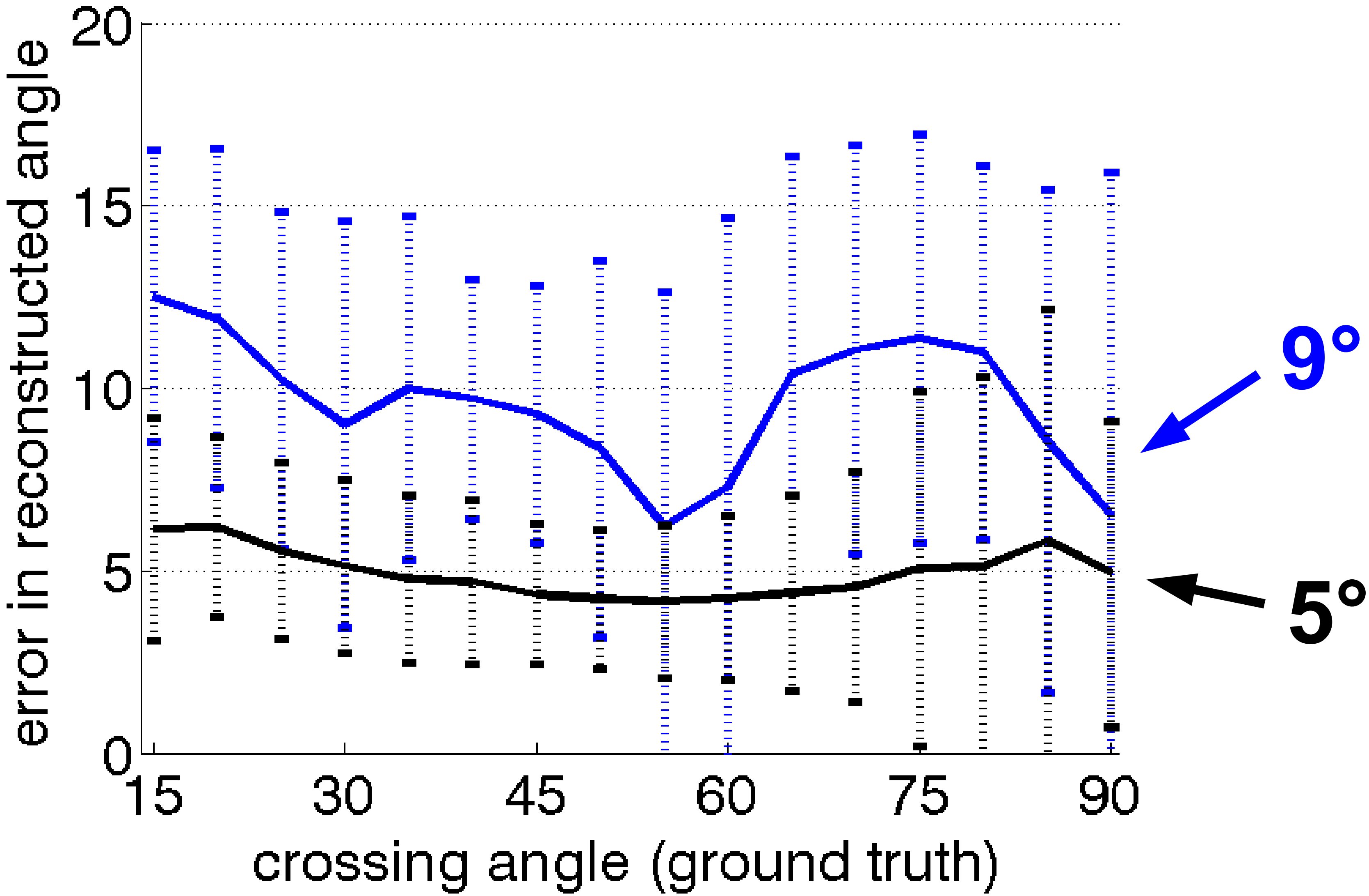
- two-fiber model
- unscented Kalman filter

signal reconstruction error



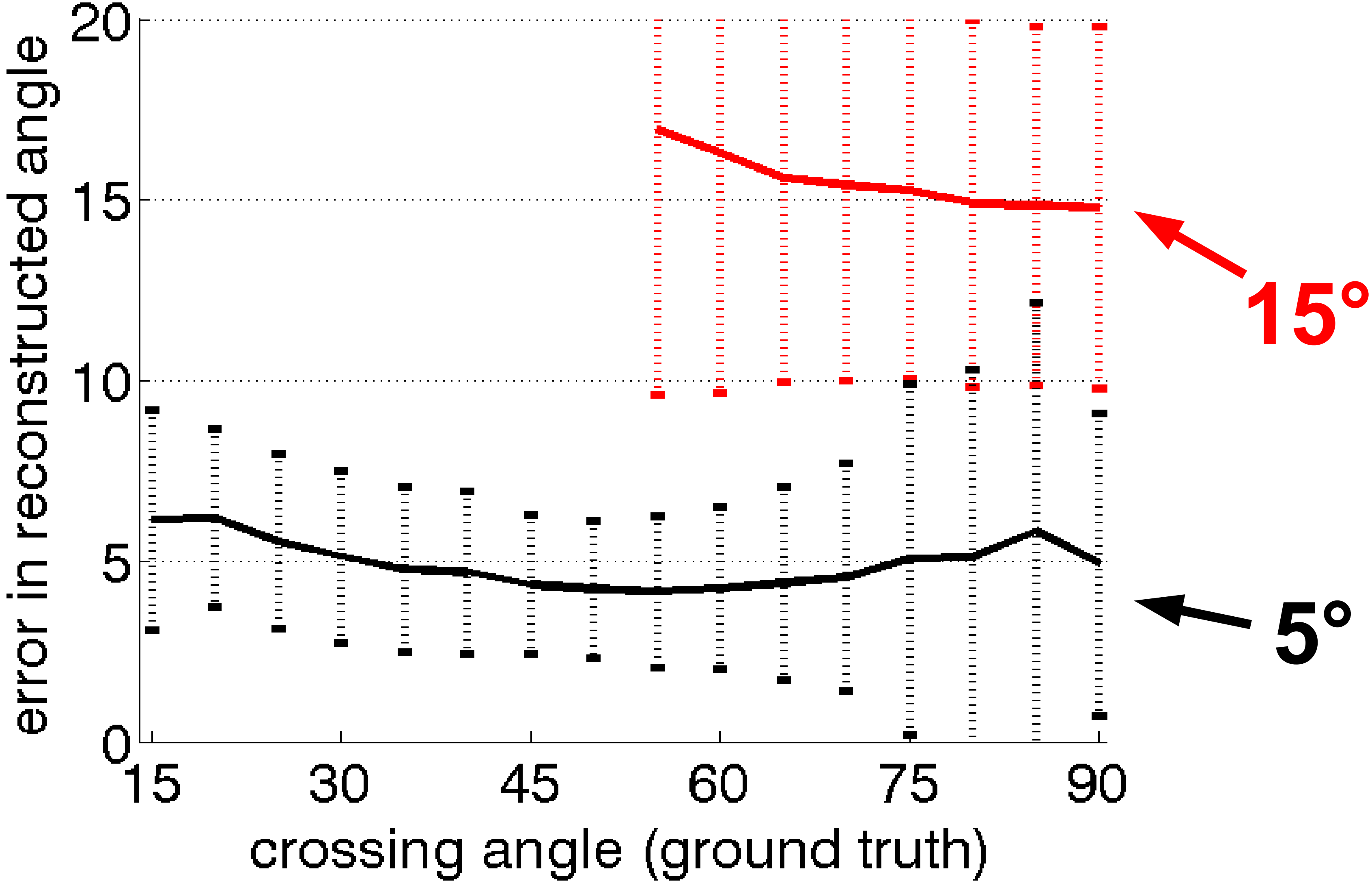
SNR ≈ 5, b = 1000

angular reconstruction error



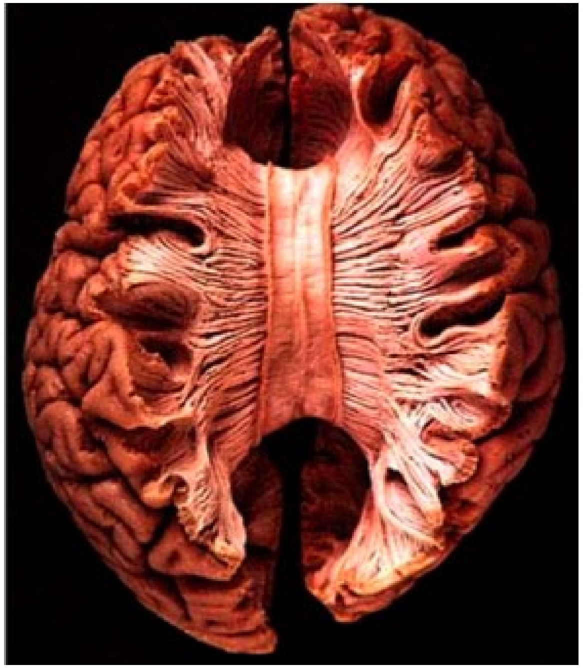
SNR ≈ 5, b = 1000

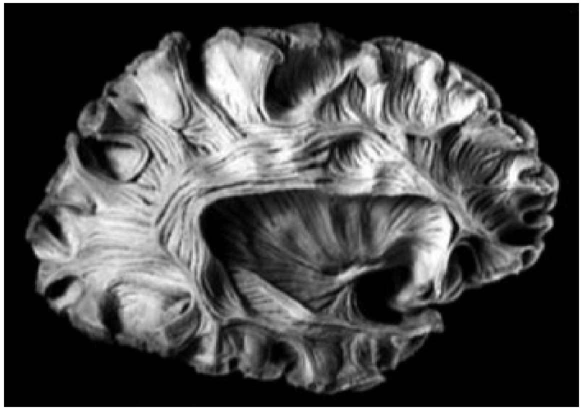
angular reconstruction error



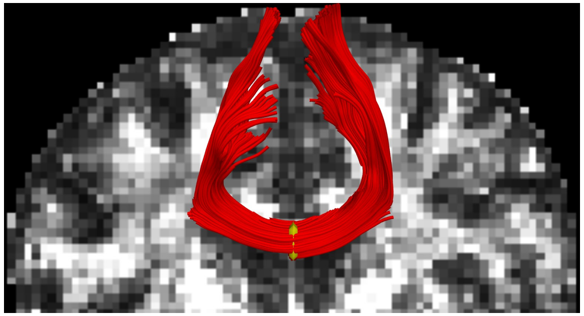
SNR ≈ 5, b = 1000

in vivo

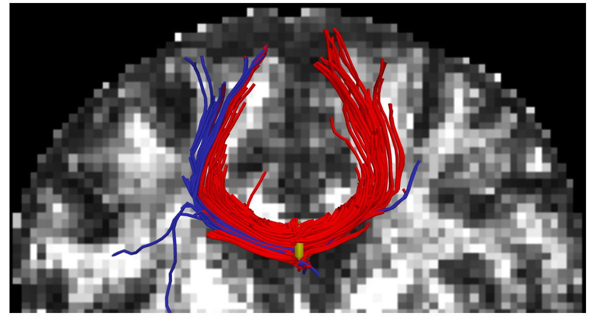




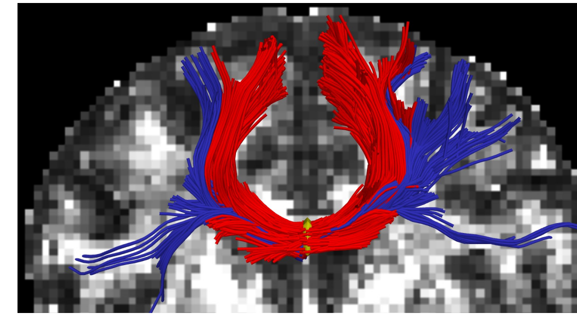
b=900
51 directions
1.7mm isotropic voxel
17 minute scan



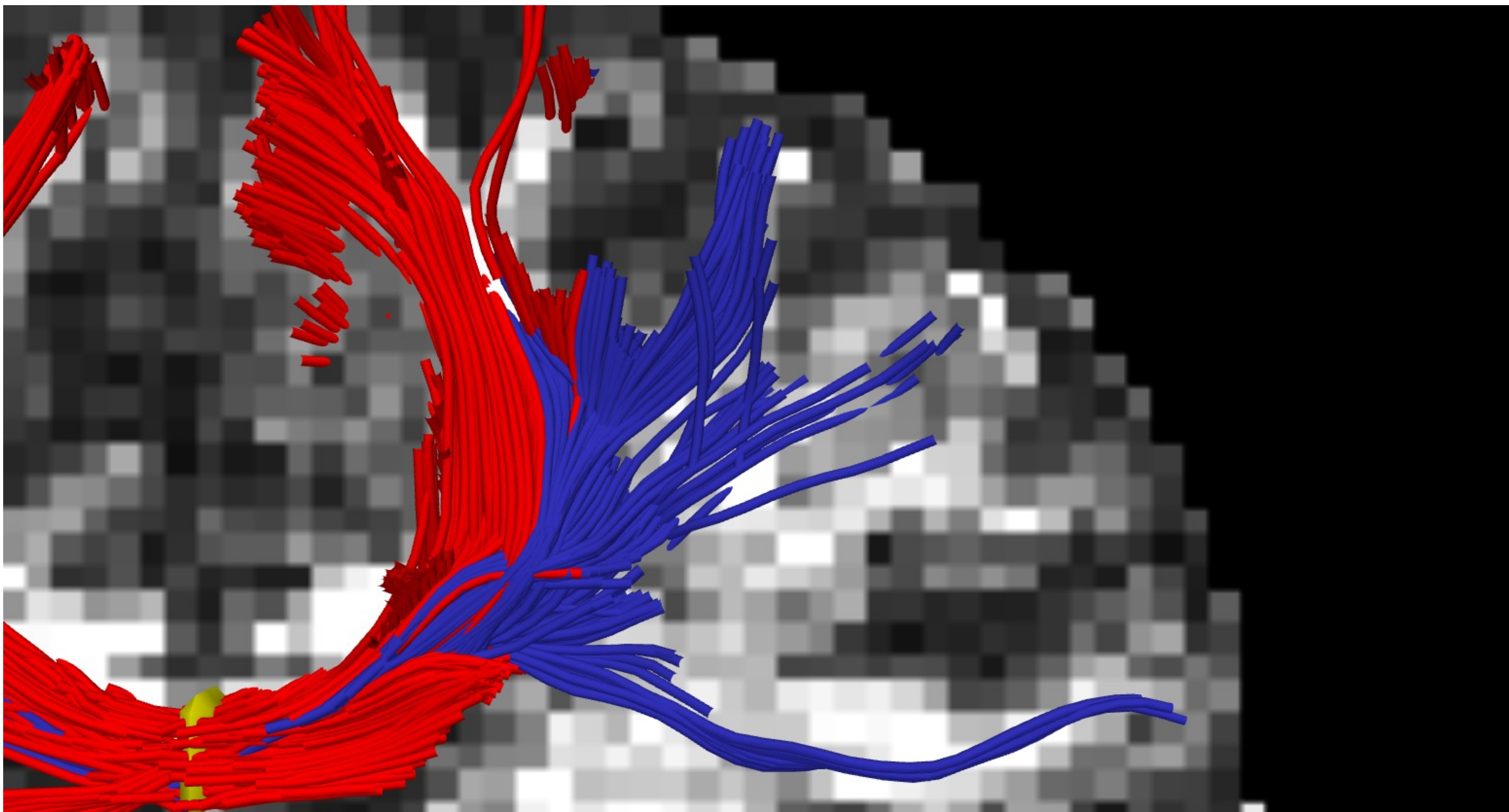
single tensor



spherical harmonics

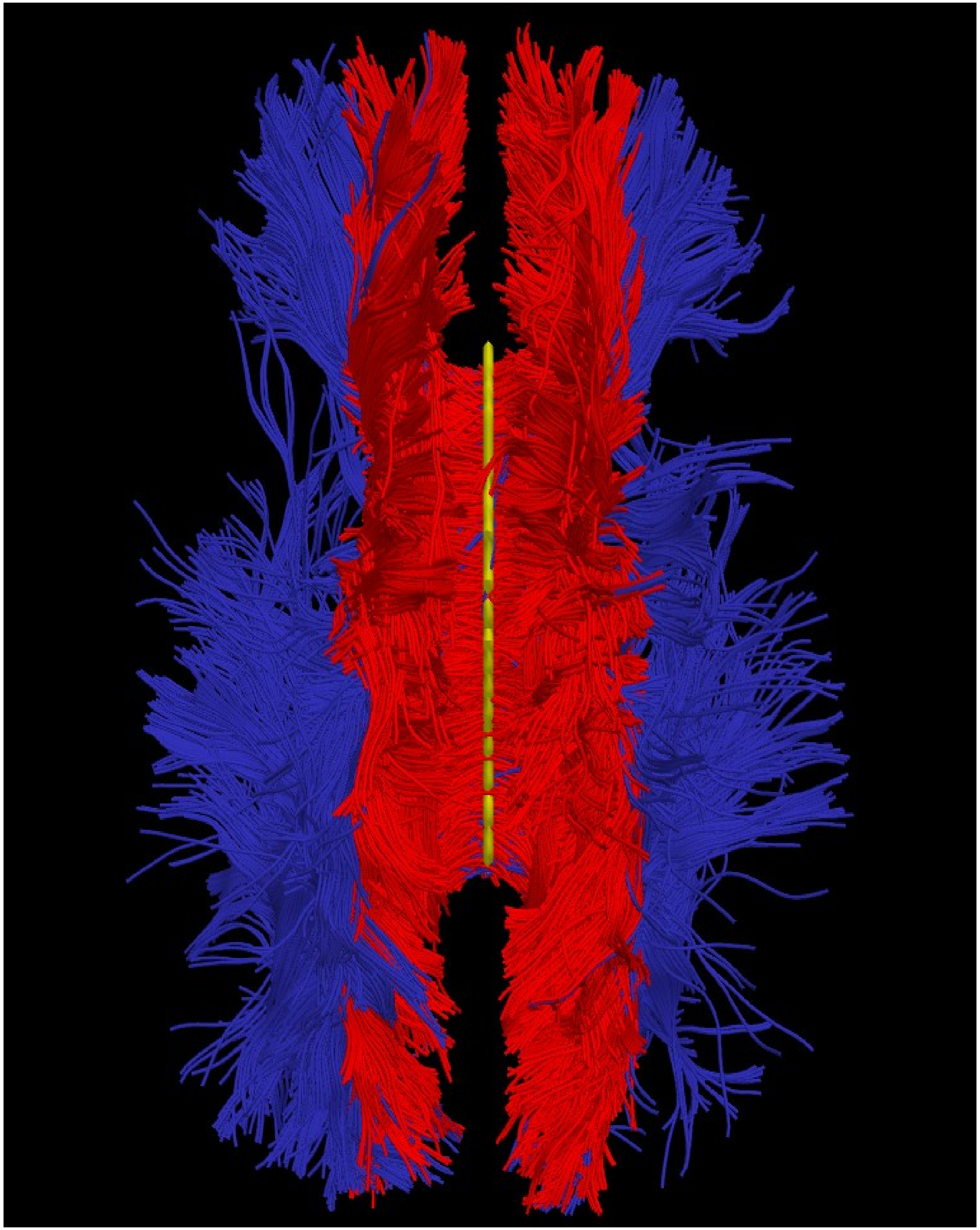


filtered two-Watson

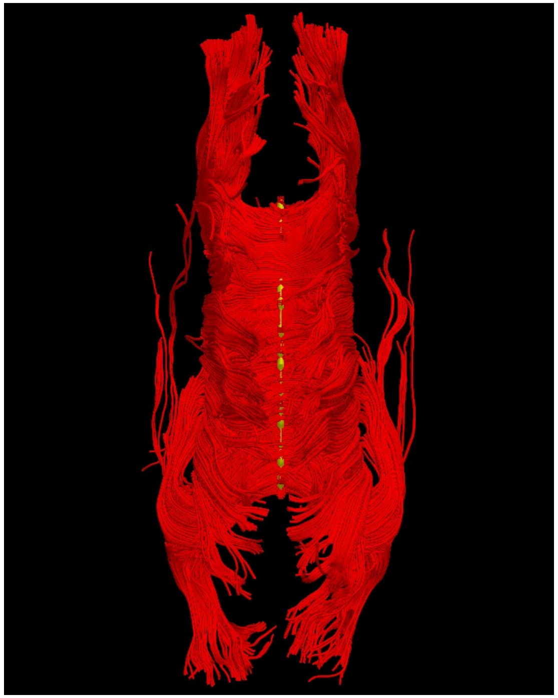


filtered two-Watson

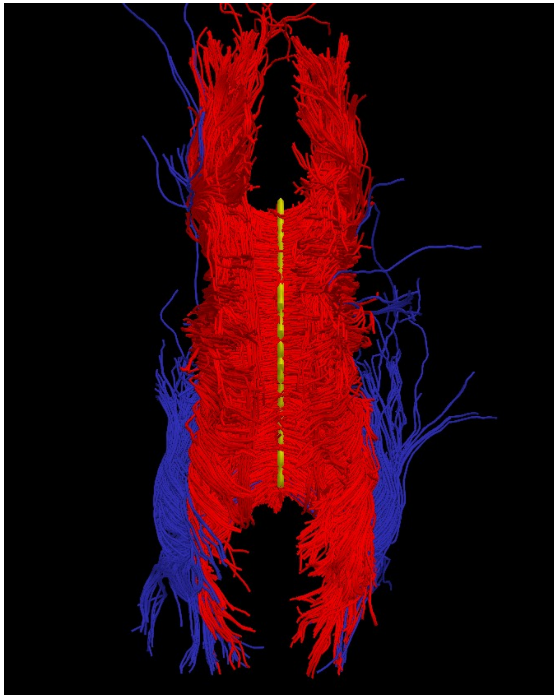
(b = 900, 1.7mm, 51 directions)



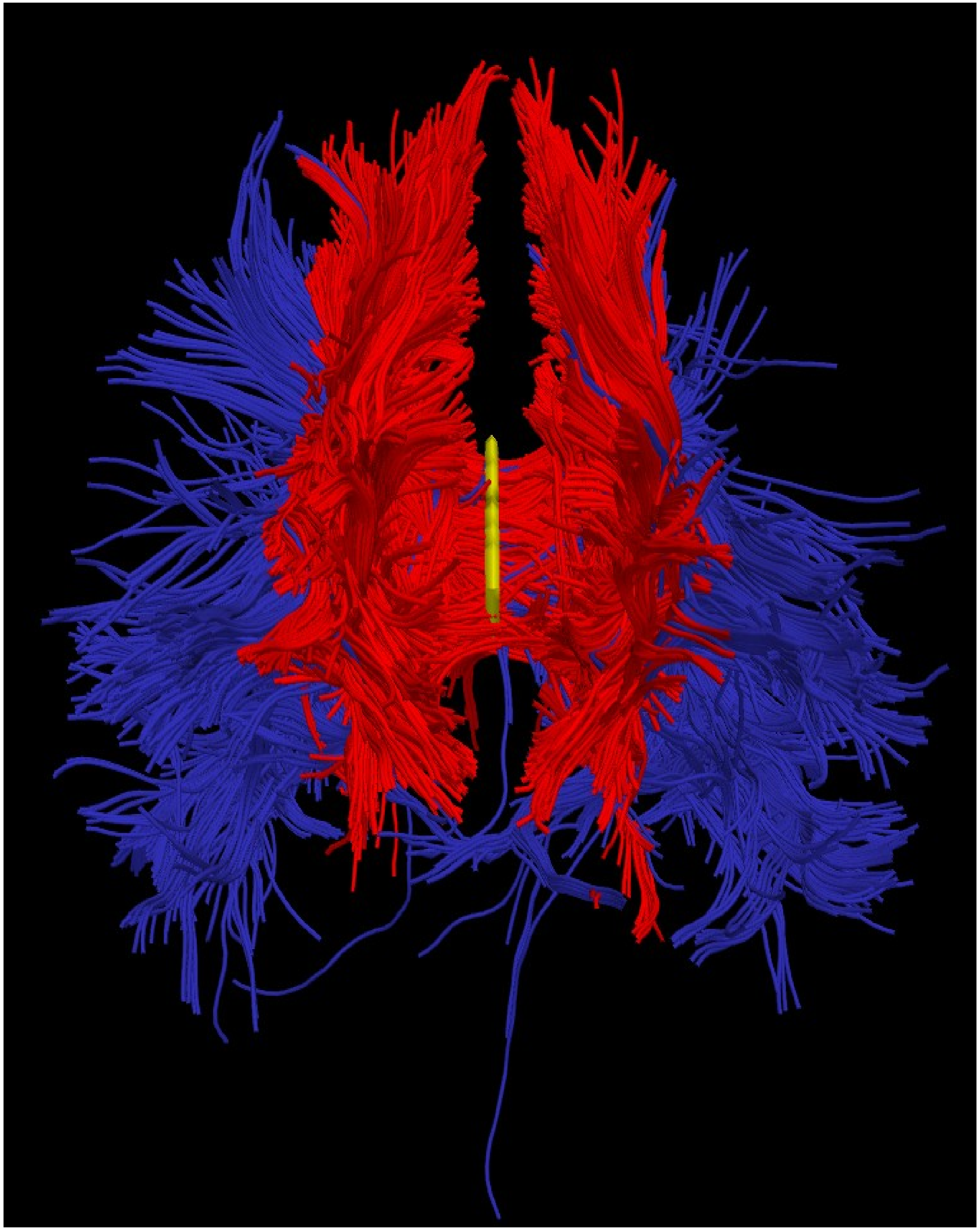
filtered two-Watson



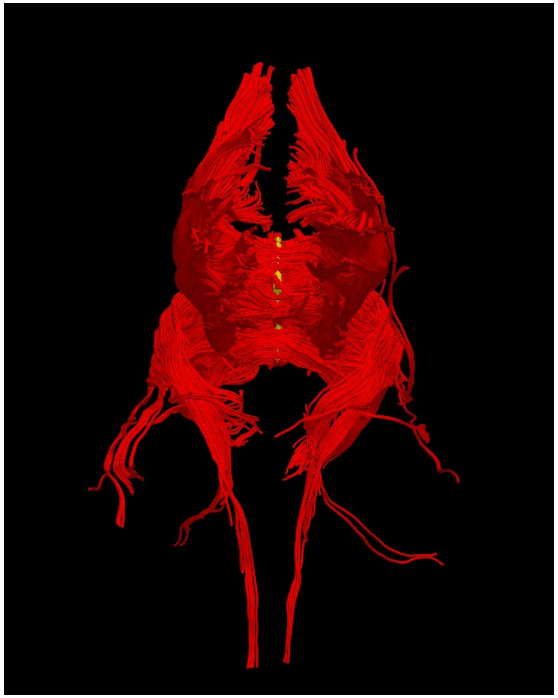
single tensor



spherical harmonics

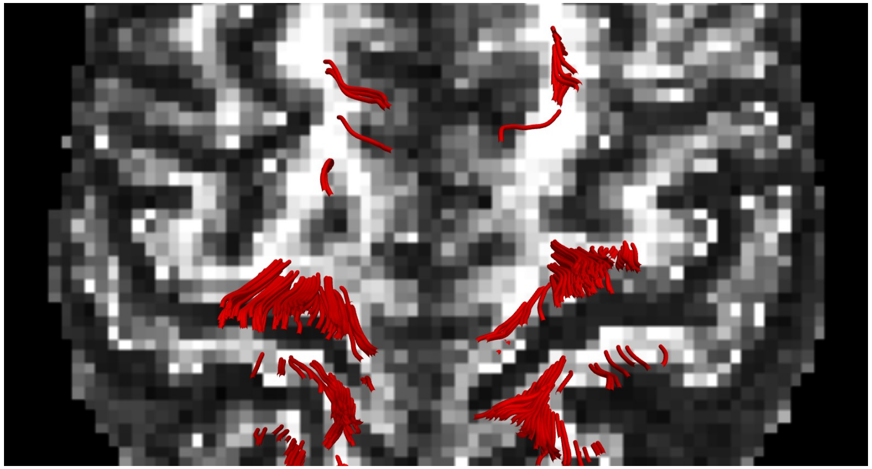


filtered two-Watson

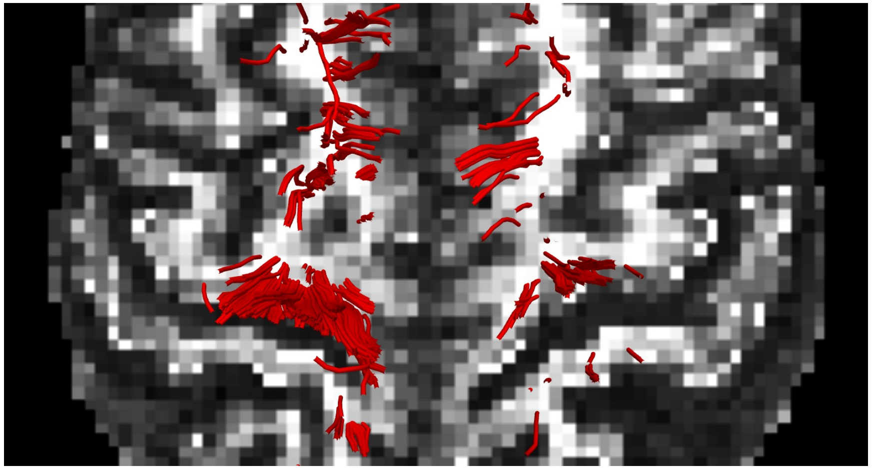


single tensor

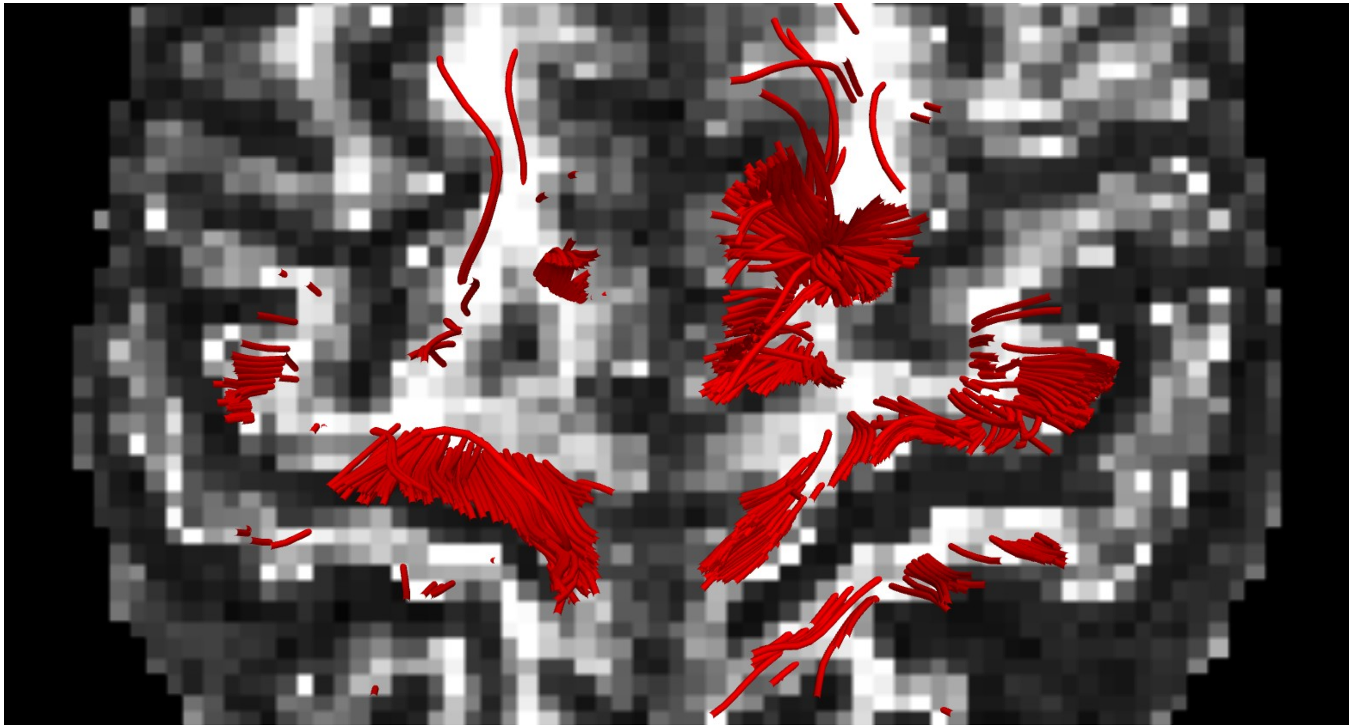
spherical harmonics



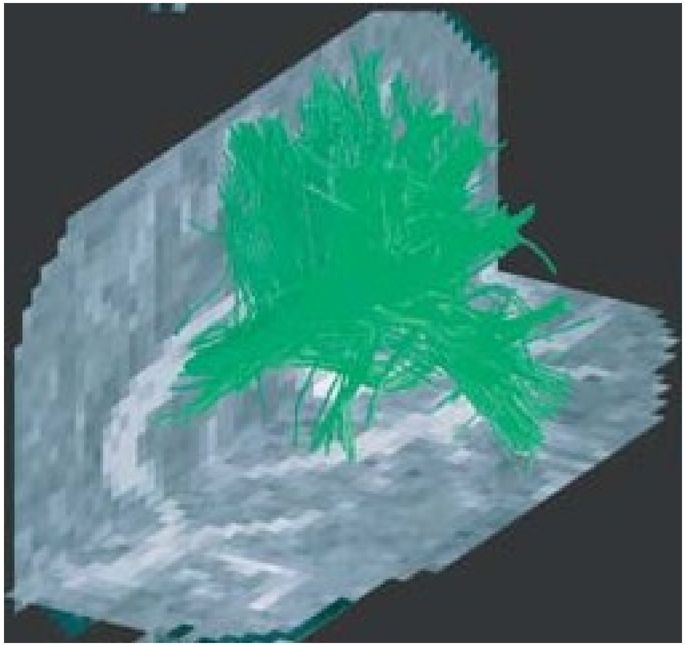
single tensor

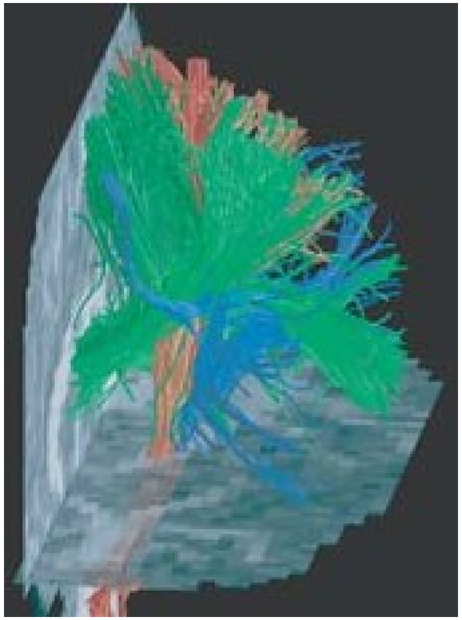


spherical harmonics

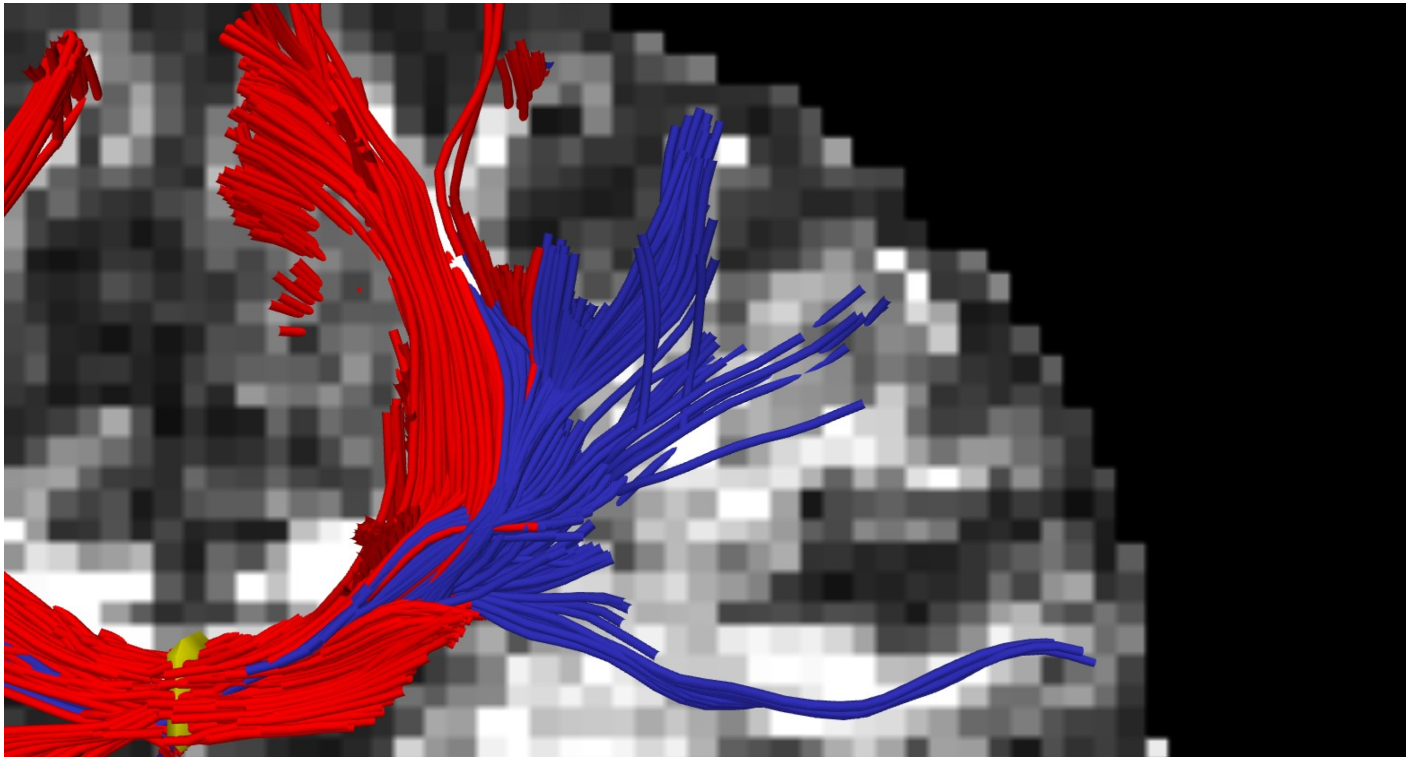


filtered two-Watson

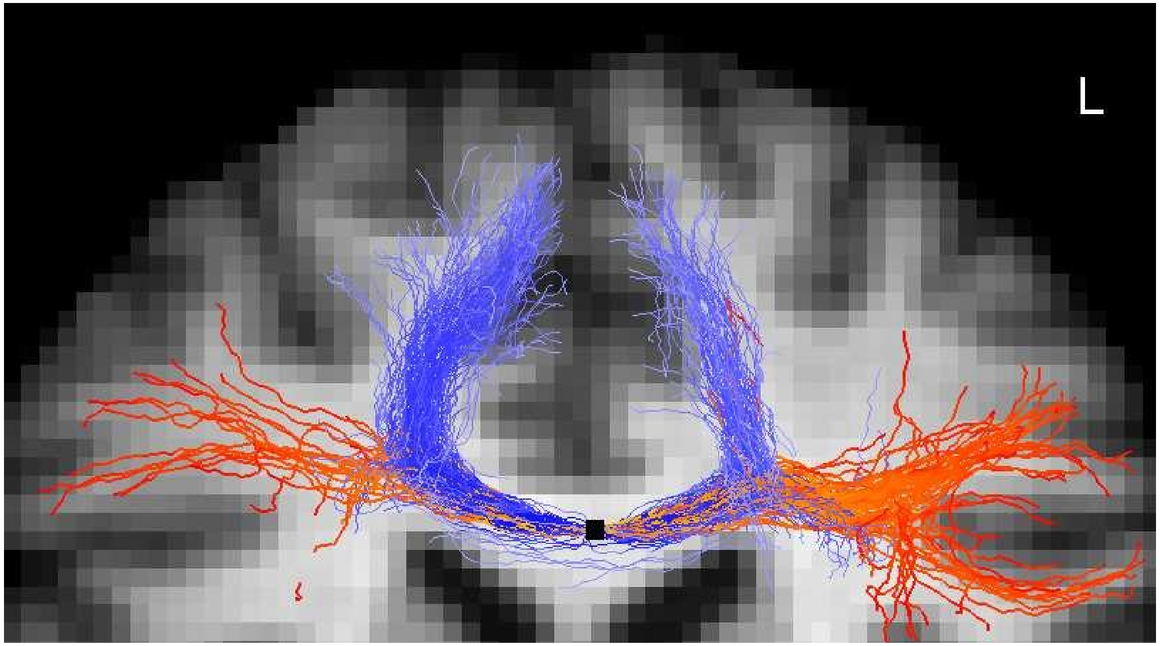




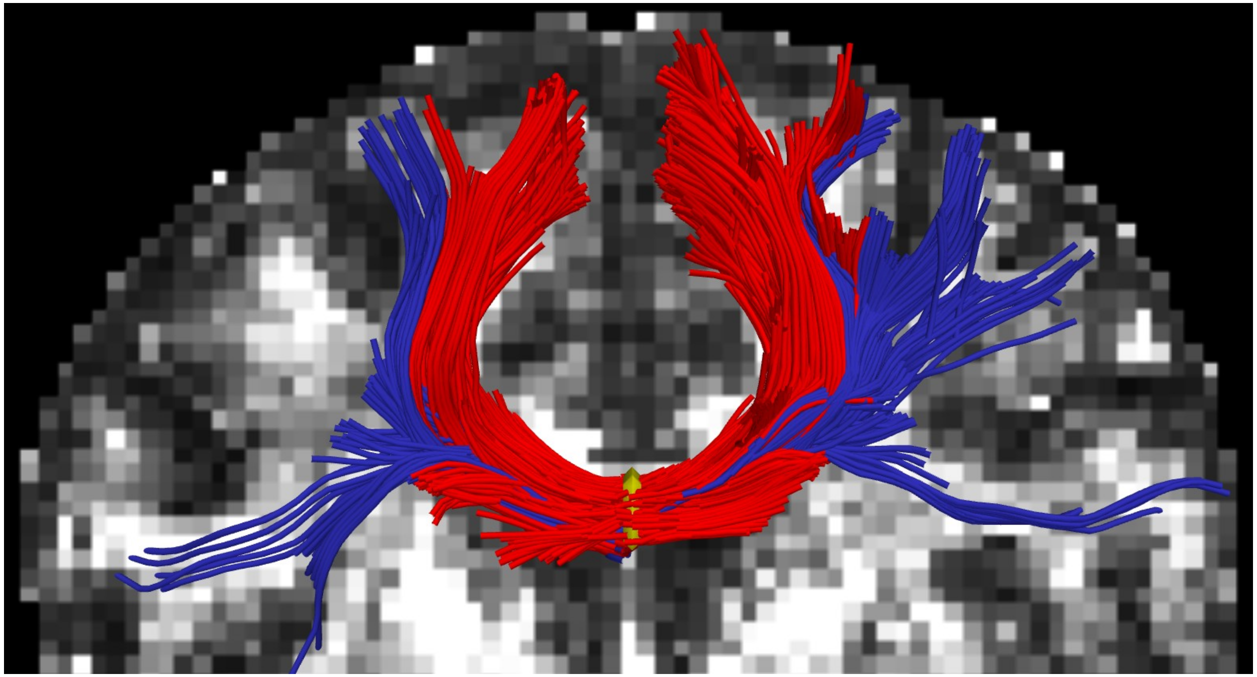
DSI [Hagmann 05]



filtered two-Watson



spherical harmonics, **probabilistic** tractography
[Descoteaux 2009]



filtered two-Watson

model assumptions

...in this proposal

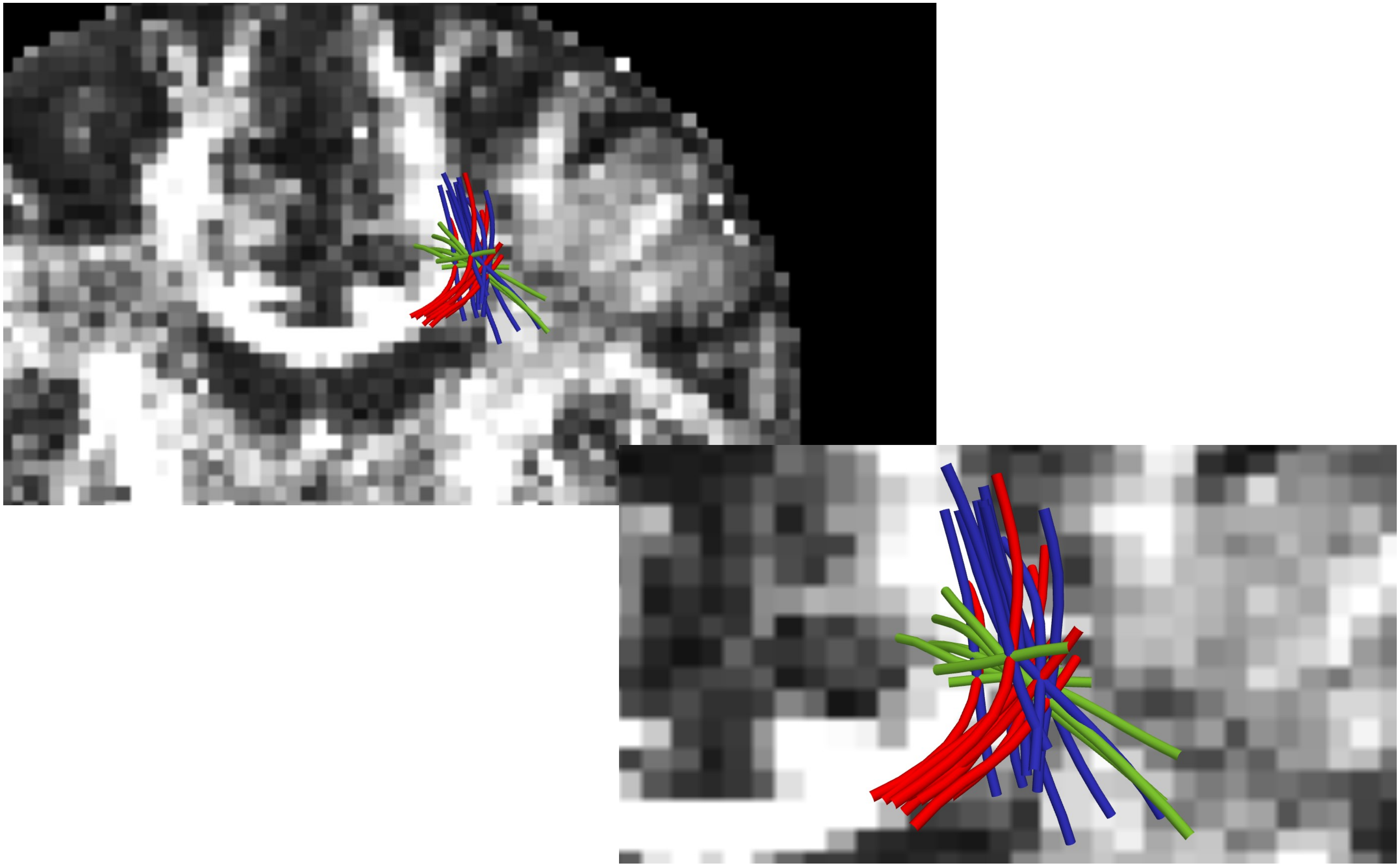
Two fibers

single fiber: align

Fixed volume fractions

scaling factor

model selection, non-parametric, ...



Algorithm 1 Unscented Kalman Filter

1: Form weighted sigma points $\mathbf{X}_t = \{w_i, \mathbf{x}_i\}_{i=0}^{2n}$ around current mean $\mathbf{x}_t$ and covariance P_t with scaling factor ζ

$$\mathbf{x}_0 = \mathbf{x}_t \qquad \mathbf{x}_i = \mathbf{x}_t + [\sqrt{\zeta P_t}]_i \qquad \mathbf{x}_{i+n} = \mathbf{x}_t - [\sqrt{\zeta P_t}]_i$$

2: Predict the new sigma points and observations

$$\mathbf{X}_{t+1|t} = f[\mathbf{X}_t] \qquad \mathbf{Y}_{t+1|t} = h[\mathbf{X}_{t+1|t}]$$

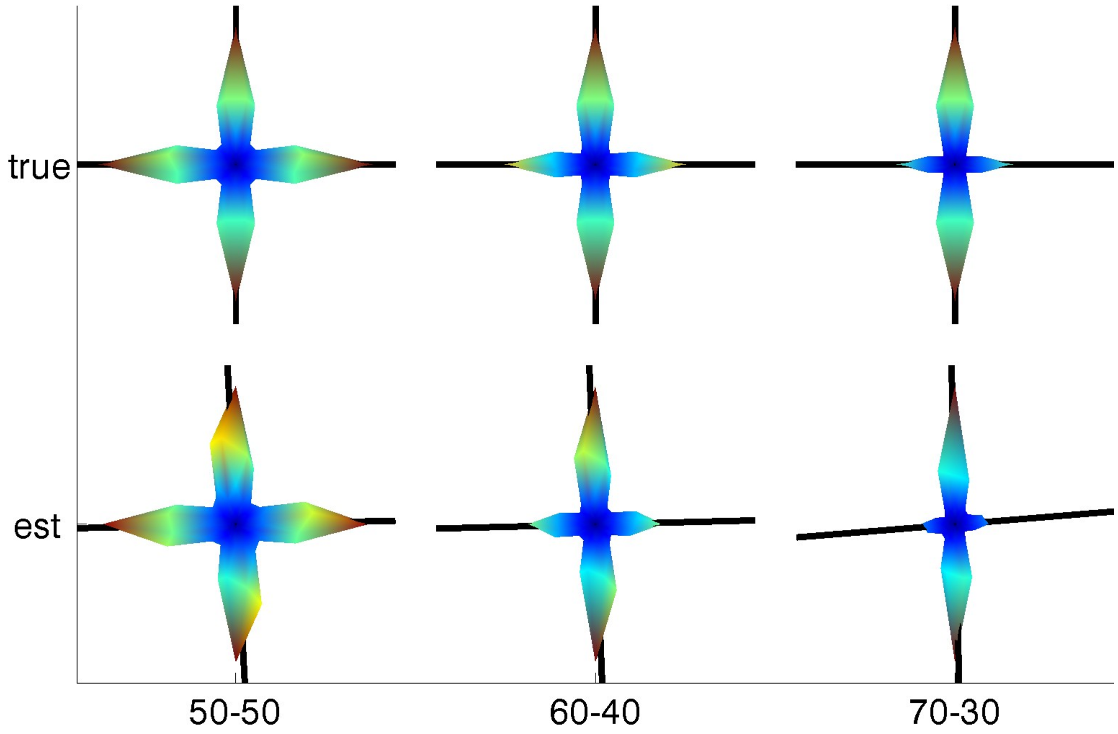
3: Compute weighted means and covariances, *e.g.*

$$\bar{\mathbf{x}}_{t+1|t} = \sum_i w_i\, \mathbf{x}_i \qquad P_{xy} = \sum_i w_i(\mathbf{x}_i - \bar{\mathbf{x}}_{t+1|t})(\mathbf{y}_i - \bar{\mathbf{y}}_{t+1|t})^T$$

4: Update estimate using Kalman gain K and scanner measurement $\mathbf{y}_t$

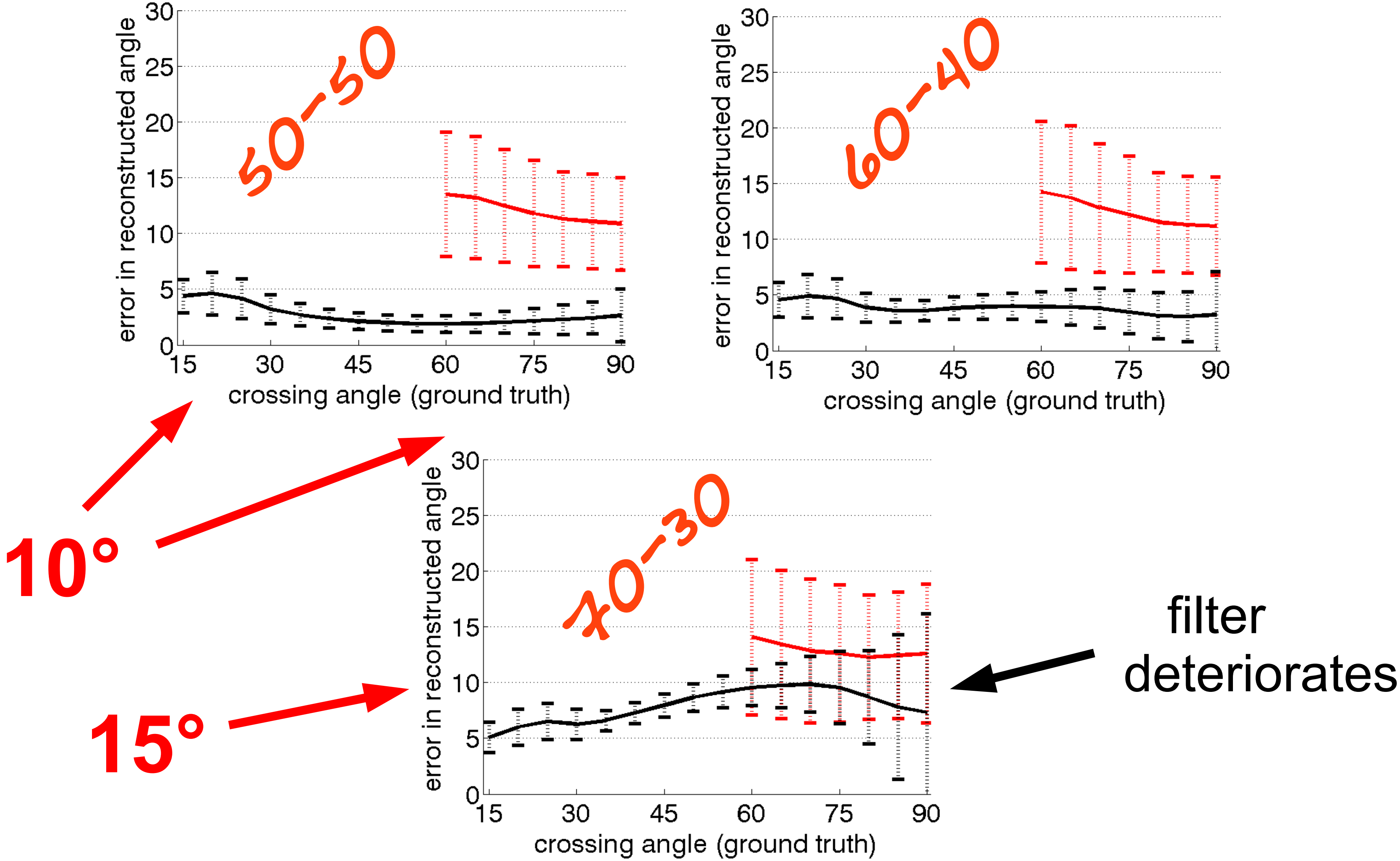
$$\mathbf{x}_{t+1} = \bar{\mathbf{x}}_{t+1|t} + K(\mathbf{y}_t - \bar{\mathbf{y}}_{t+1|t}) \qquad P_{t+1} = P_{xx} - KP_{yy}K^T \qquad K = P_{xy}P_{yy}^{-1}$$

volume fractions

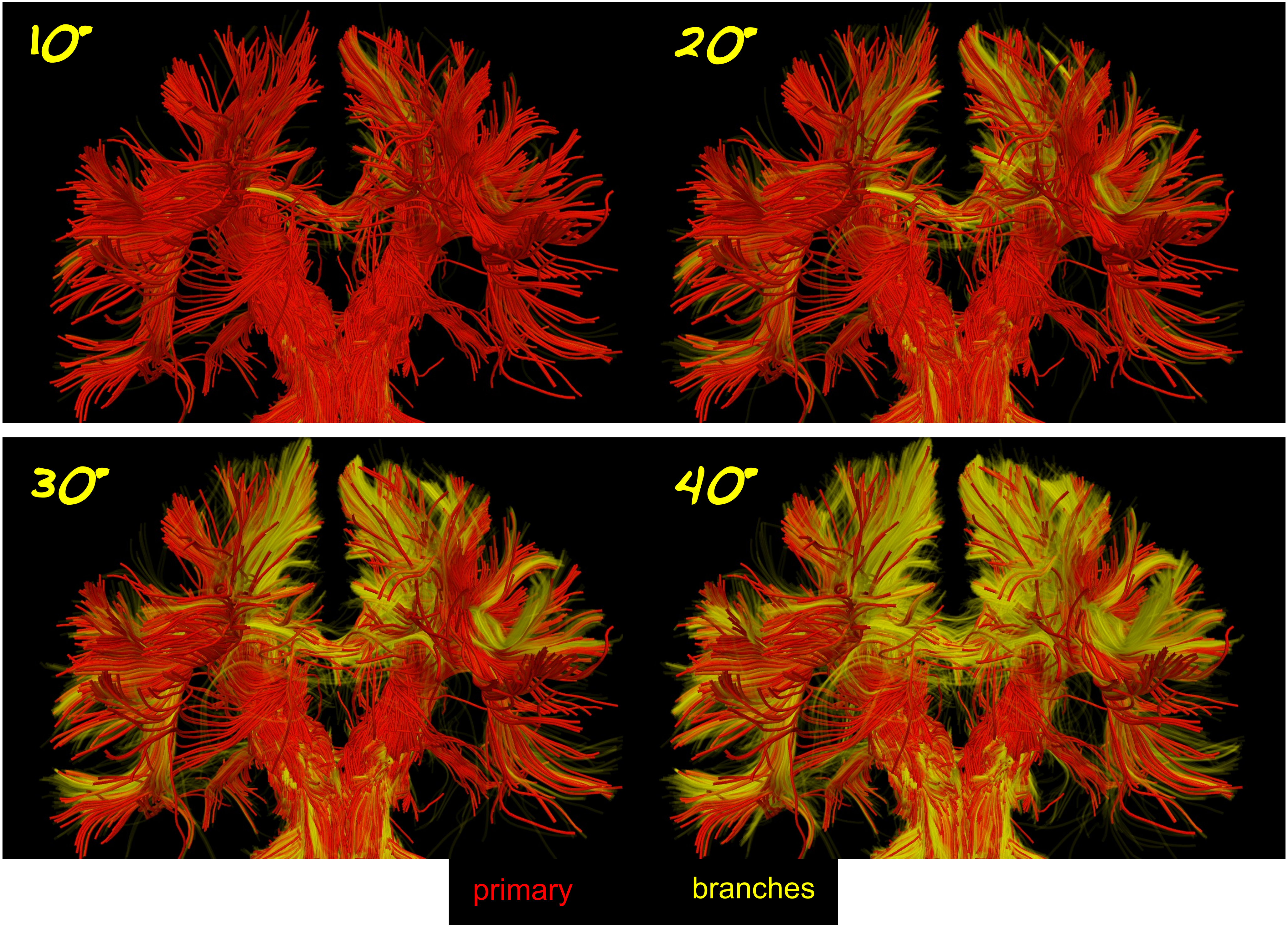


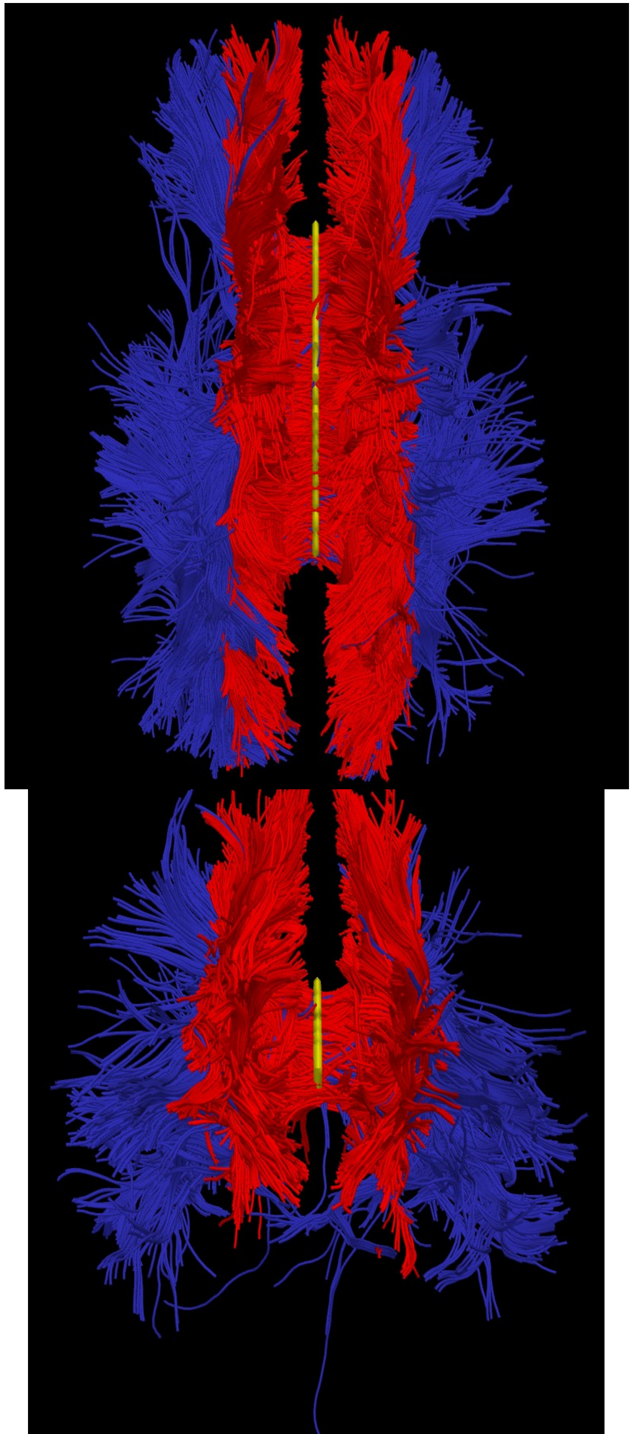
SNR ≈ 10, b = 1000

volume fractions



SNR ≈ 10, b = 1000

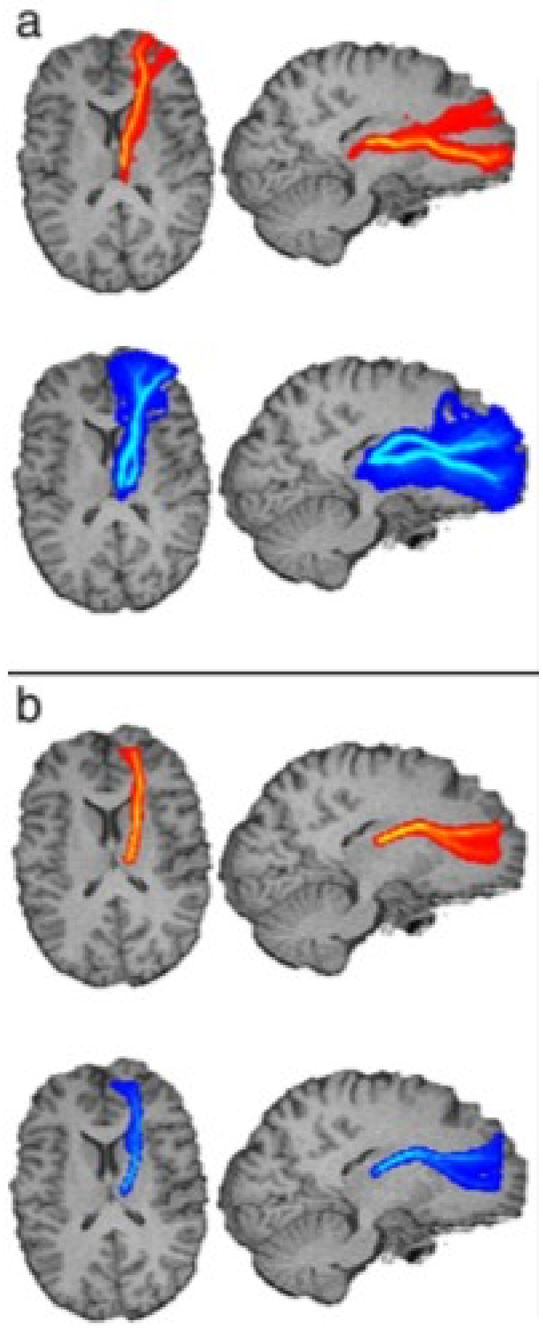

10°
20°
30°
40°
primary
branches



filtered two-Watson

DSI (Hagmann 2005)

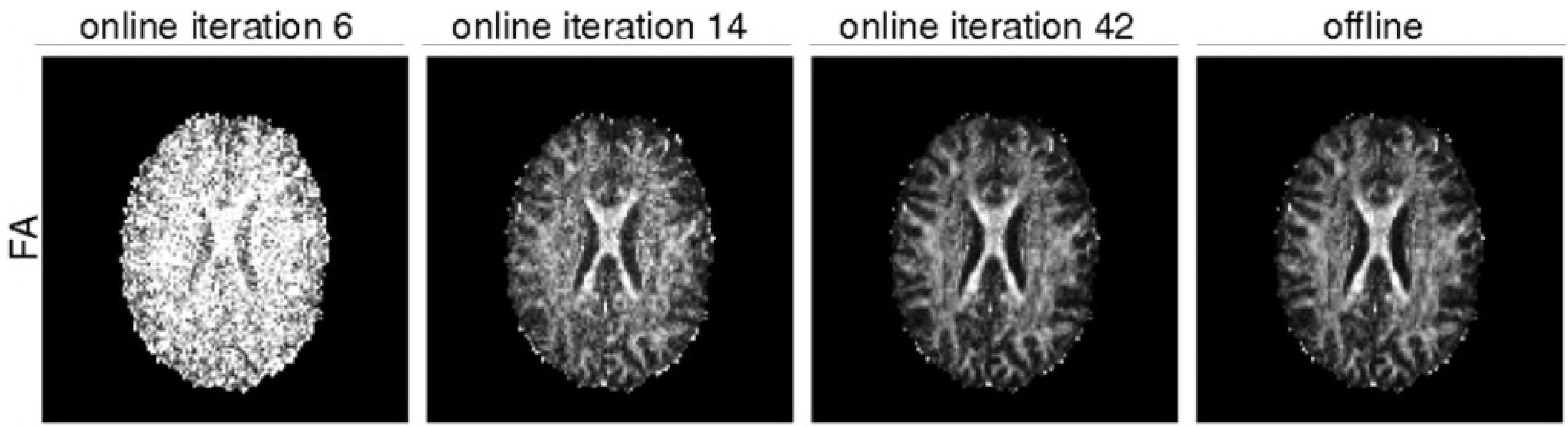
DSI (Hagmann 2005)



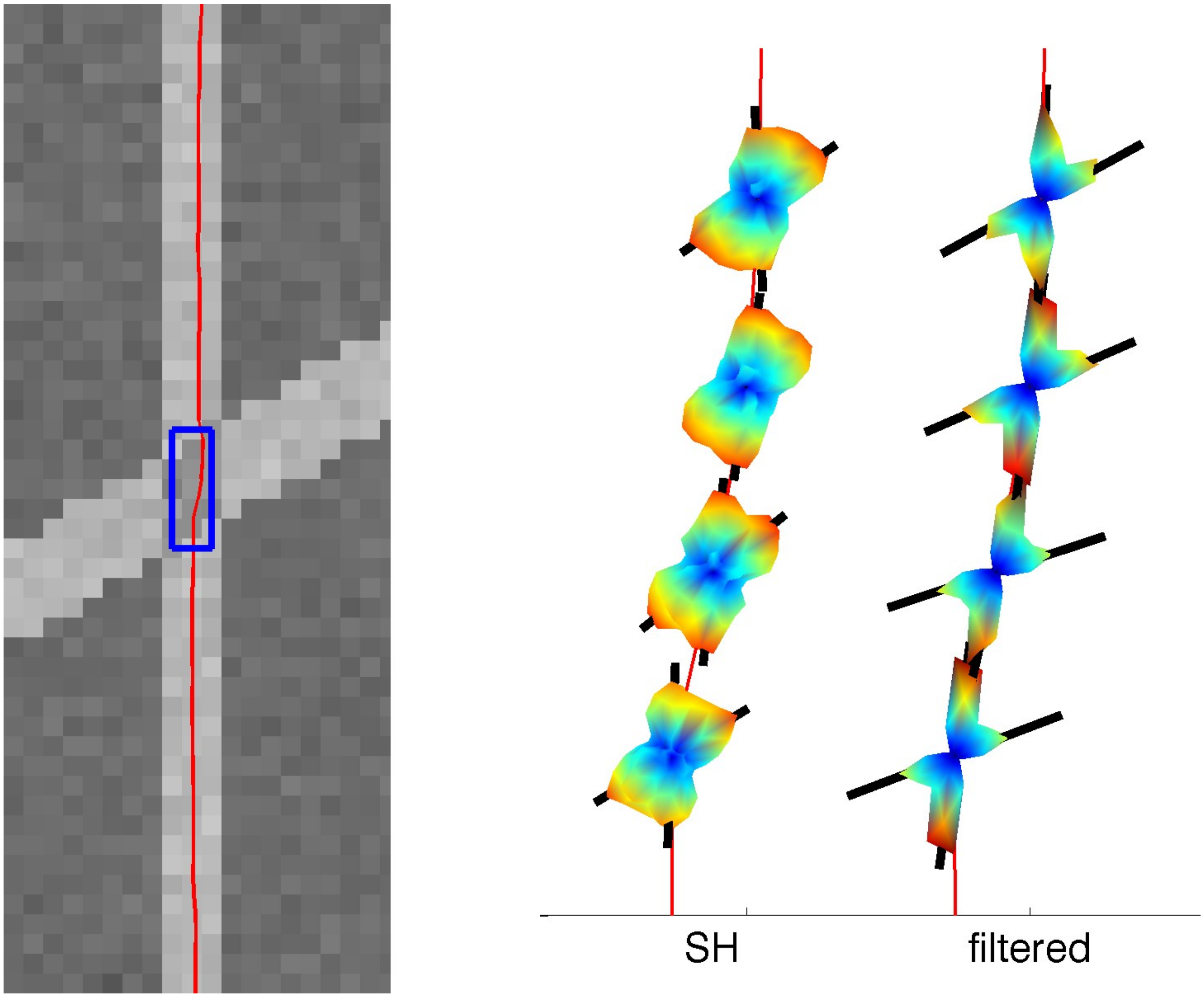
[Jababdi 07]

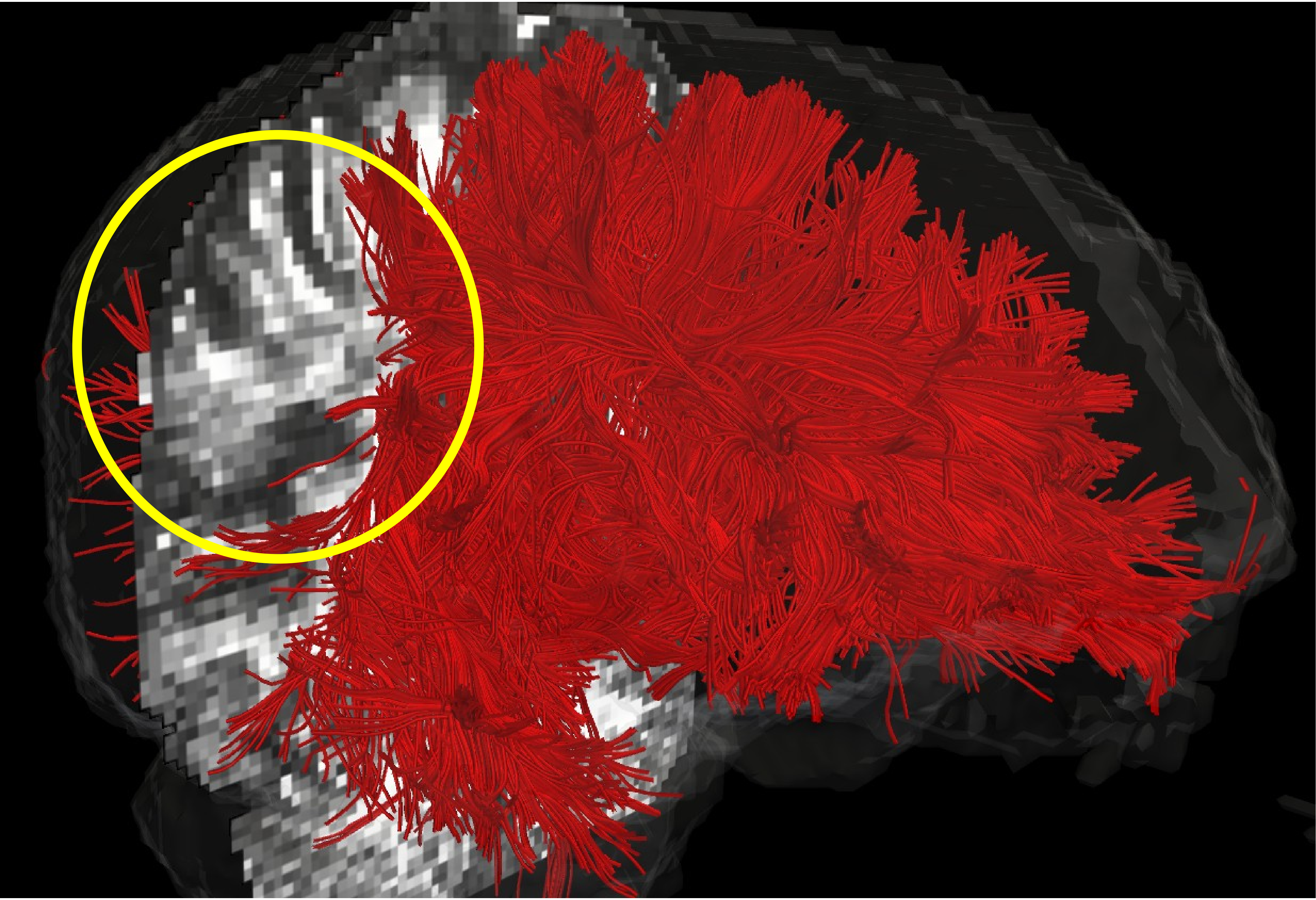
ROI-based

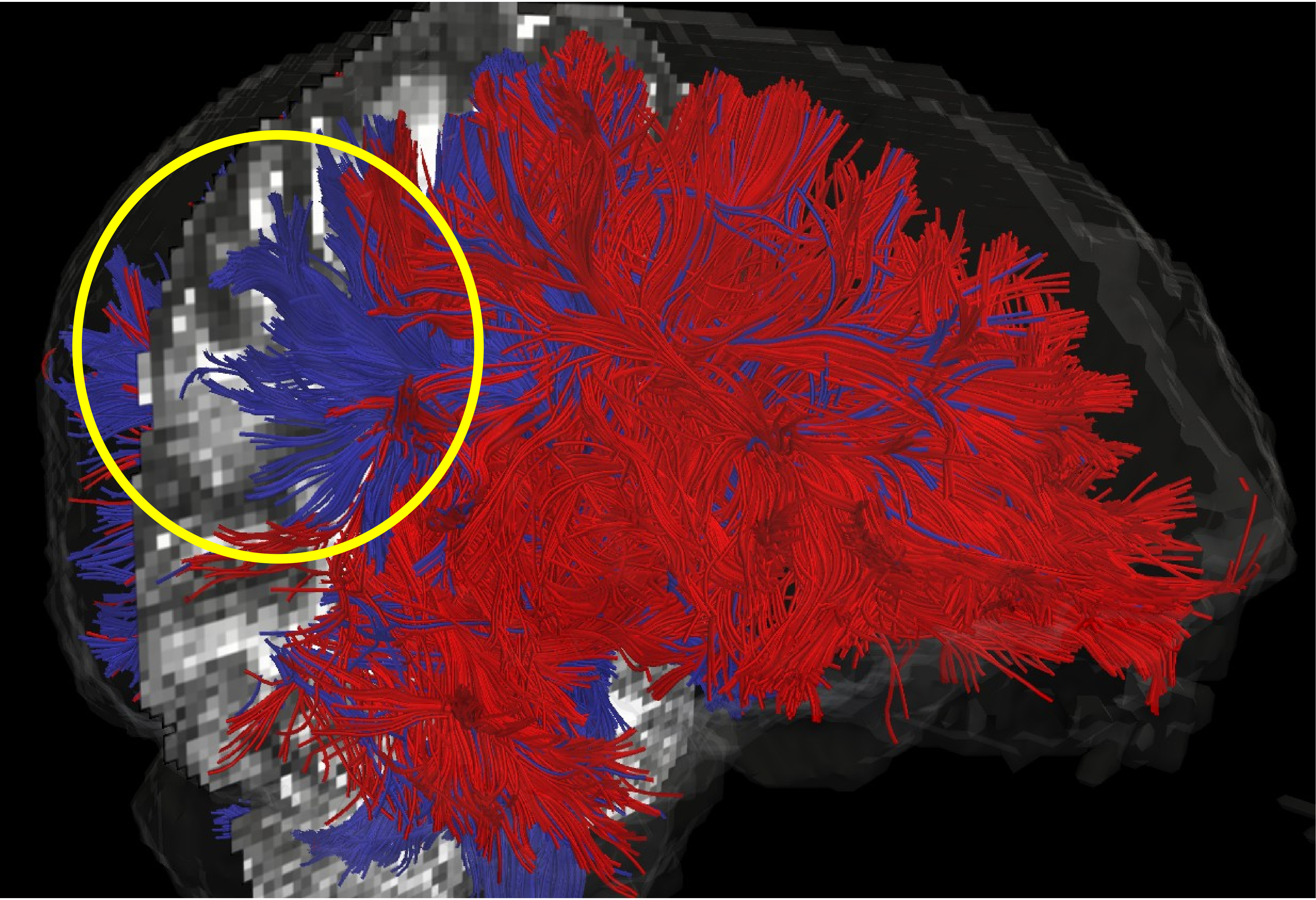
to constrain
probabilistic

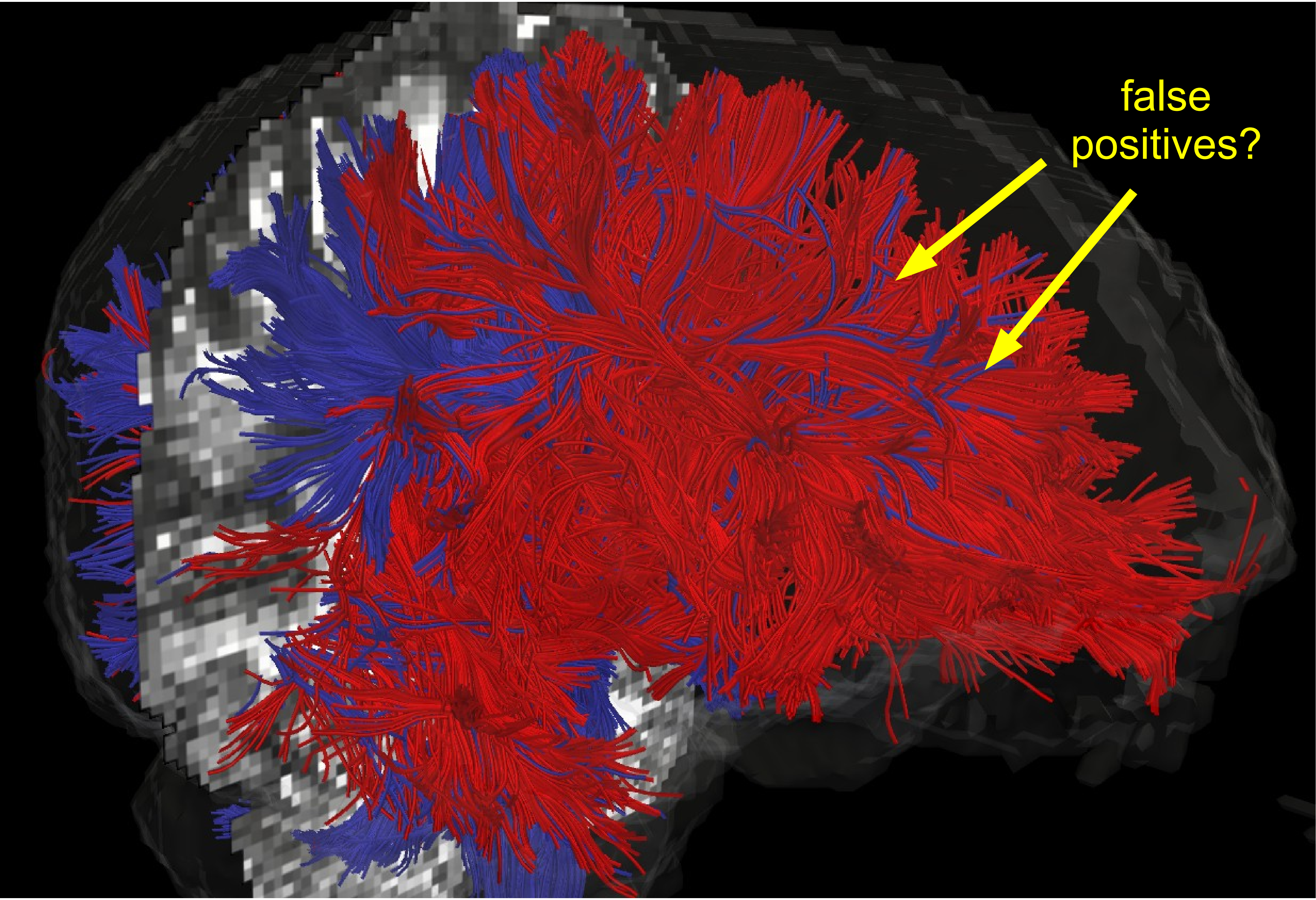


[Poupon 08,Deriche 09]

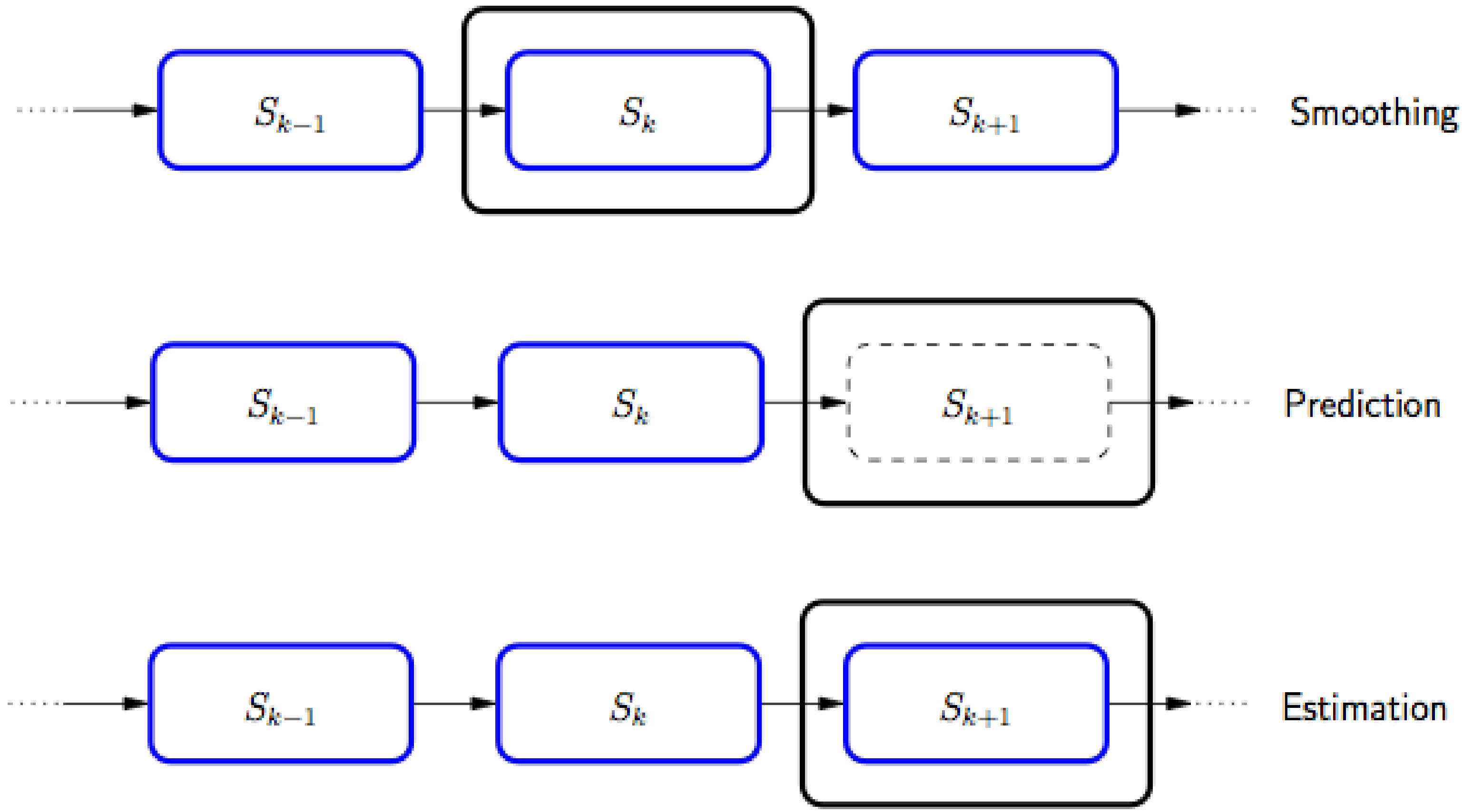

SH
filtered






false
positives?

filtering



[Niethammer 08]

brute force optimization

- matching pursuit
- parametric dictionary
- noiseless signal
- discretization, noise

spherical harmonics

- non-parametric
- order eight (8)
- fiber sharpening for peak detection (L=0.006)

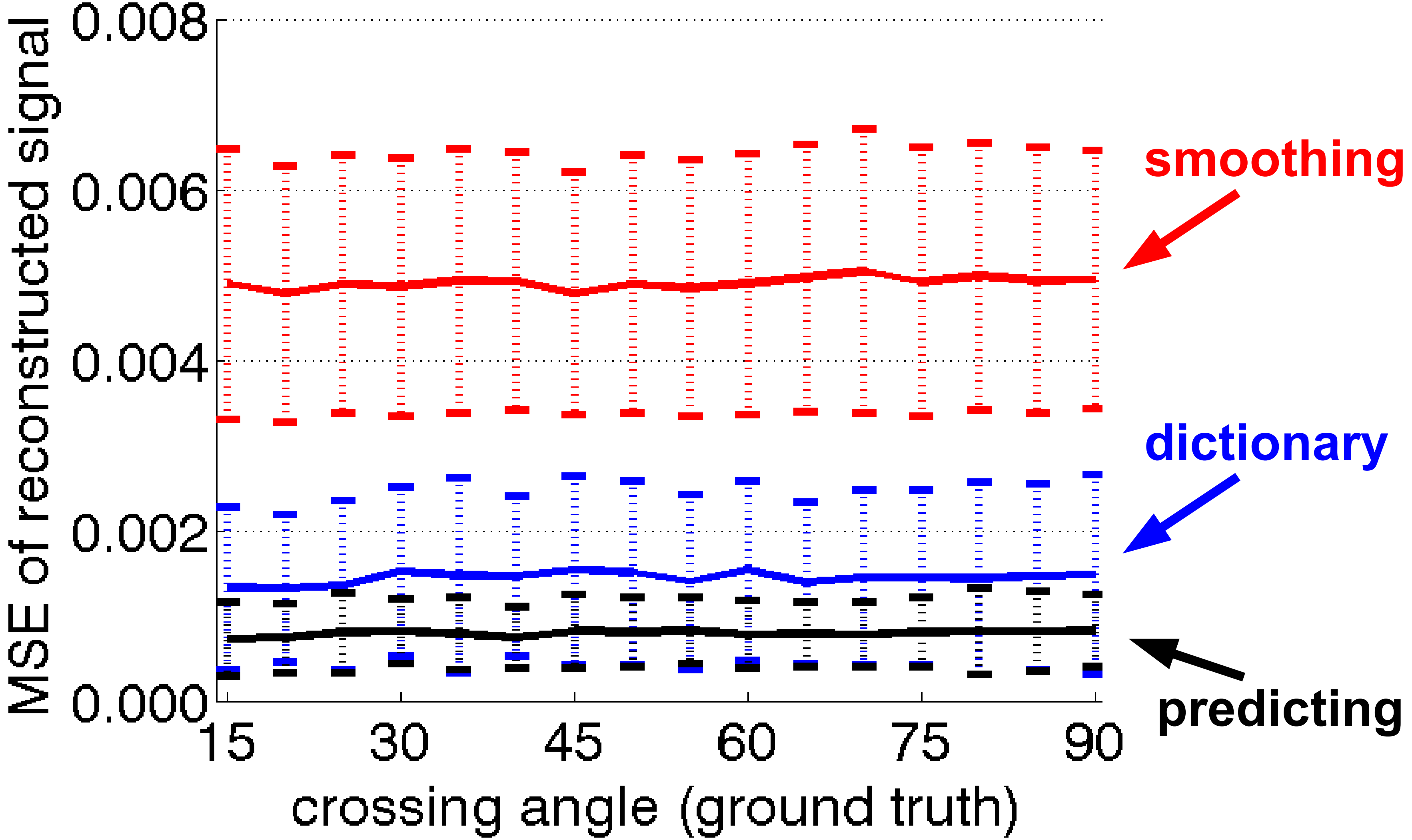
[Descoteaux 07]

filtered tractography

- two-fiber model
- unscented Kalman filter

J. G. Malcolm, M. E. Shenton, and Y. Rathi. Neural tractography using an unscented Kalman filter. In *Information Processing in Medical Imaging (IPMI)*, pages 126–138, 2009.

signal reconstruction error



SNR ≈ 5, b = 1000