# MIXTURE MODEL FOR ESTIMATING FIBER-ODF AND MULTI-DIRECTIONAL TRACTOGRAPHY

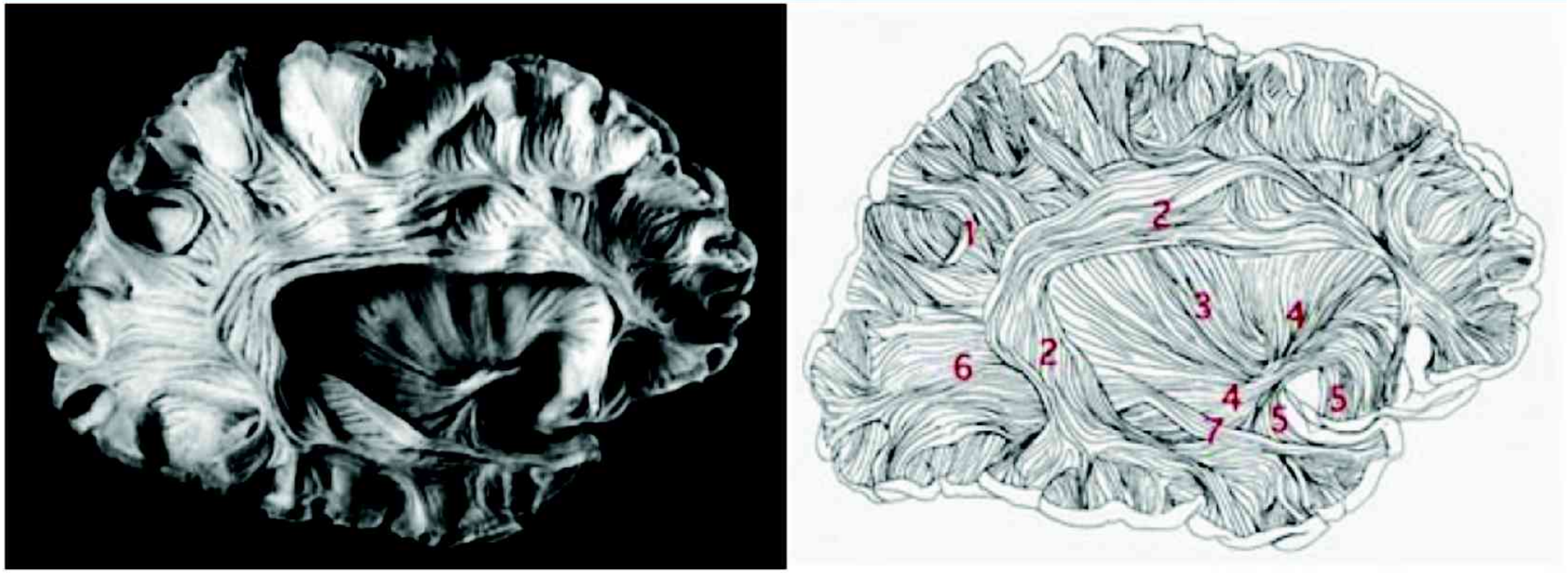
Y. Rathi, J. G. Malcolm, S. Bouix, G. Kindlmann, C-F. Westin, M. Kubicki and M. E. Shenton

Harvard Medical School, Boston, MA

# OUTLINE

- Background – Q-ball imaging
- Existing methods
- Directional Functions (Watson)
- Mixture Model Estimation
- Tractography on in-vivo data

# BACKGROUND – ANATOMY OF THE BRAIN

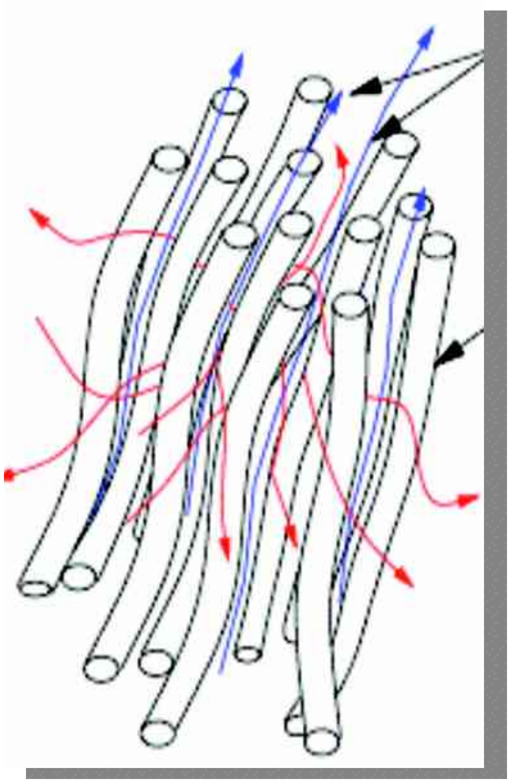


1. Short arcuate bundles. 2. Superior longitudinal fasciculus. 3. External capsule. 4. Inferior occipitofrontal fasciculus. 5. Uncinate fasciculus. 6. Sagittal stratum. 7. Inferior longitudinal fasciculus.

Short and long association fibers in the right hemisphere
[Williams et.al. 97]

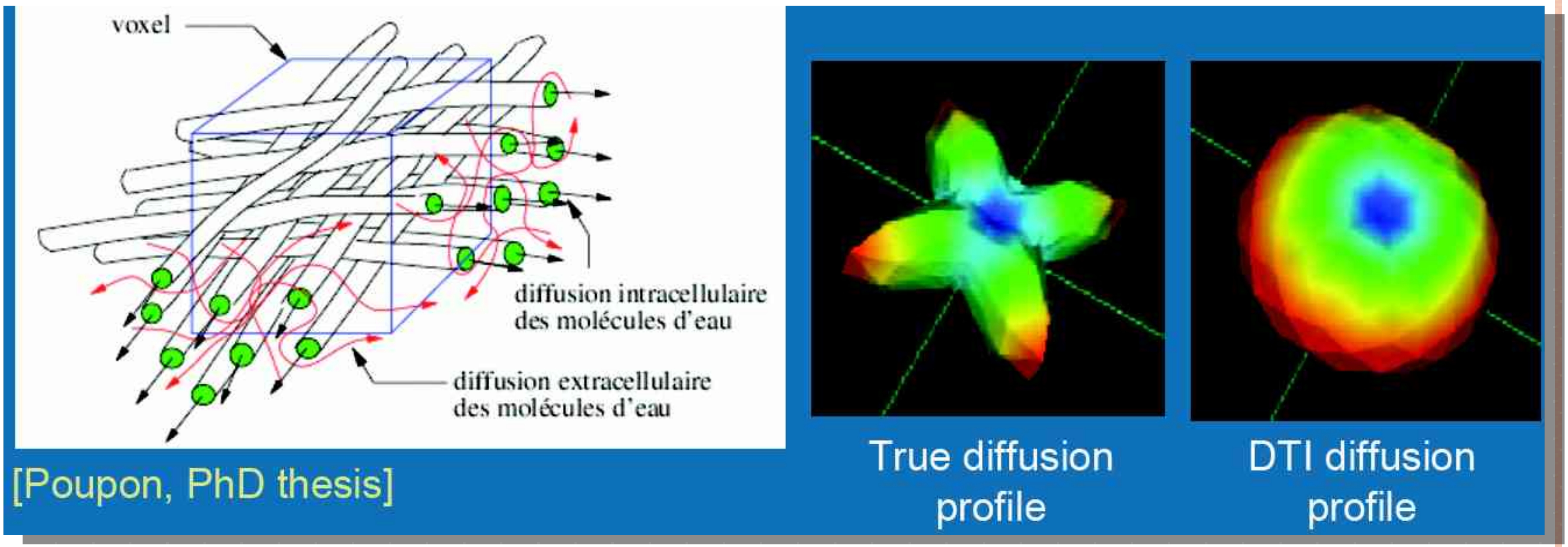
# BACKGROUND – DIFFUSION MRI

- Brownian motion or average PDF of water molecules is along white matter fibers.
- Signal attenuation is proportional to average diffusion in each voxel.



Poupon Ph.D. Thesis

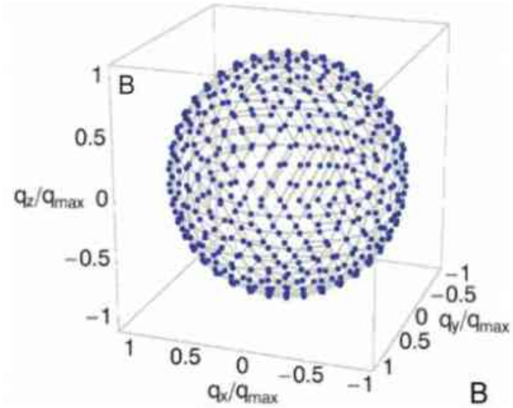
# BACKGROUND – LIMITATION OF DTI




DTI fails to capture complex diffusion patterns i.e., multi-fiber orientations

# BACKGROUND - HARDI

- High Angular Resolution Diffusion Imaging (HARDI) – acquire data from multiple directions by sampling points on the sphere.



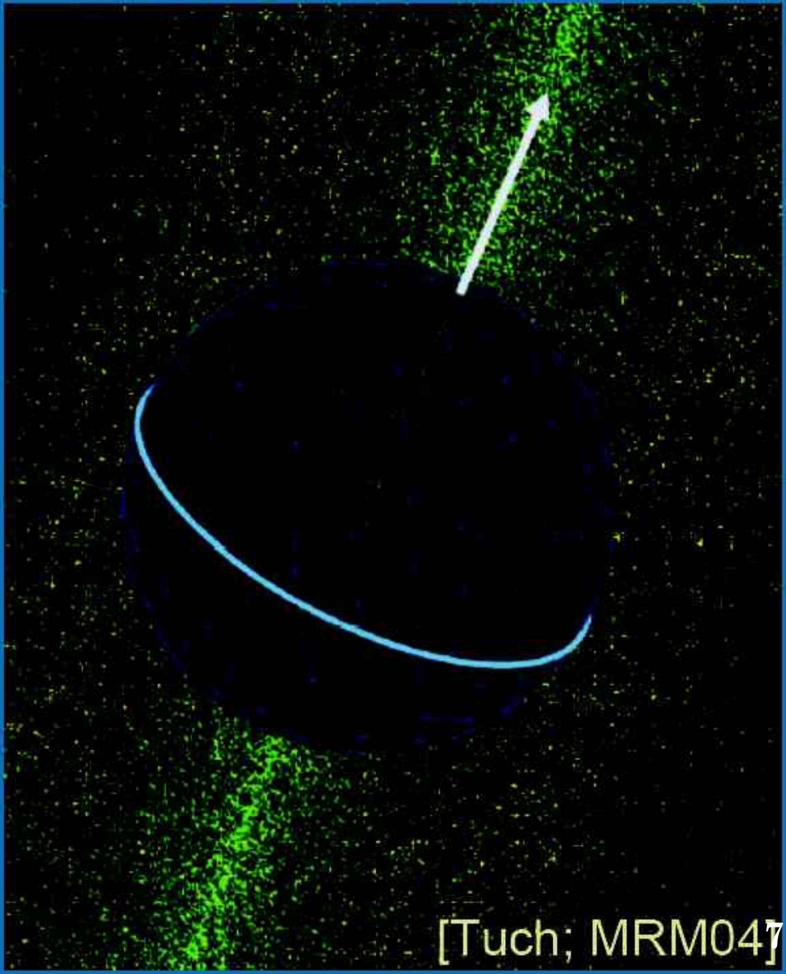

- Recover multi-fiber orientations
- Use Funk-Radon transform to estimate the diffusion ODF (dODF).

# ODF ESTIMATION – FUNK RADON TRANSFORM

- ODF can be computed directly from the HARDI signal over a single ball
- Integral over the perpendicular equator

- Funk-Radon Transform

$$\mathcal{G}[f(\mathbf{w})](\mathbf{u}) = \int \delta(\mathbf{u}^T\mathbf{w})f(\mathbf{w})d\mathbf{w}$$



# EXISTING METHODS

- Tuch [Q-Ball imaging] proposed to use spherical radial basis functions to estimate the ODF ==> Requires a large number of coefficients to represent a single dODF (N sampling directions = N coefficients).
- Alexander et. al., Descoteaux et. al., use spherical harmonics (SH) to represent dODF's. A SH of order 6 or 8 requires 28 or 45 coefficients to represent a dODF.
- Other methods include variants of spherical deconvolution, high-order tensors and mixture of Gaussian or Wishart distributions.

# DIRECTIONAL FUNCTIONS

- We propose to model the measured signal using a mixture of the Watson directional function defined on the sphere, under the following assumptions:

$$S(\mathbf{u}_i) = S_i = S_0 \sum_{j=1}^{k} w_j \exp(-b\mathbf{u}_i^T \mathbf{D}_j \mathbf{u}_i) + n_i$$

If we assume, each component represents one principal diffusion direction, then the exponent in the above eqn can be approximated by:

$$-b\mathbf{u}^T D\mathbf{u} \approx -b\lambda_1 \mathbf{u}^T(\mathbf{m}\mathbf{m}^T)\mathbf{u} = -b\lambda_1(\cos(\theta))^2$$

Thus, we assume that the signal produced by a single fiber is given by:

$$W(\mathbf{u}_i) = C^{-1}\exp(-k\cos^2\theta_i);\ \theta_i = \cos^{-1}(\mathbf{u}_i^T\mathbf{m}),$$

where C is a normalization constant.

# DIRECTIONAL FUNCTIONS - WATSON

- Compute the Funk-Radon Transform (FRT) of the Watson function.
- The FRT of a function f defined on the 2-sphere is given by:

$$\mathrm{R}f(\mathbf{v}) = \frac{1}{2\pi}\int_{g\in G(\mathbf{c},\mathbf{v})} f(g)dg$$

  where, G is the great circle orthogonal to v
- For the Watson function, the FRT can be computed as follows:

$$\mathrm{R}W(\mathbf{u}_i) = \frac{1}{2\pi}\int_0^{2\pi} \exp(-k[\mathbf{m}^T\mathbf{q}(\mathbf{t})]^2)dt$$

$$\text{where } \mathbf{q}(\mathbf{t}) = \{\mathbf{q}_1\cos t + \mathbf{q}_2\sin t | t \in (0, 2\pi)\}$$



*Mixture Model for tractography - Y Rathi et al.*

# ODF – WATSON FUNCTION

- After some algebraic manipulation, we obtain:

$$\mathrm{R}W(\mathbf{u}_i) = C^{-1}\exp(-\frac{k}{2}\sin^2\theta_i)\ I_0(\sqrt{a^2+b^2}).$$

Thus, the dODF is characterized by a phase shift of π/2 multiplied by $I_0$ – bessel function of order 0.

- $I_0$ is computationally expensive, so we approximte the ODF by:

$$\mathrm{R}W(\mathbf{u}_i) \approx C^{-1}\exp(-\frac{k}{2}\sin^2\theta_i).$$



*Mixture Model for tractography - Y Rathi et al.*

# ODF - WATSON

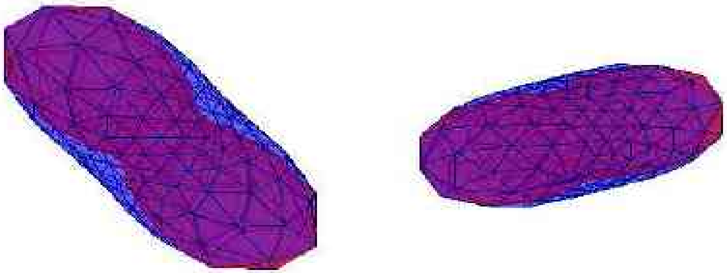


Fig. 3: The actual ODF surface (blue) and the approximaate ODF (red) for $k = 4, 3$ respectively (left to right).

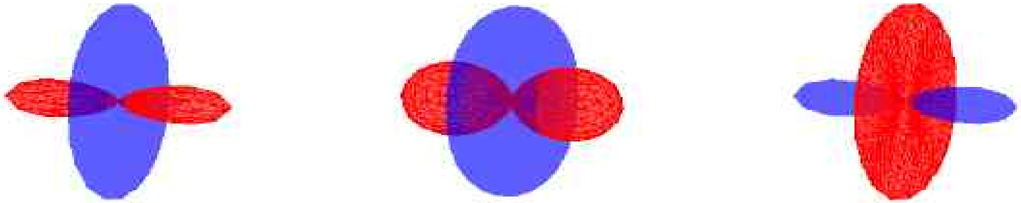


Fig. 4: The signal surface (blue) and the corresponding ODF (red) for $k = 5, 0.0001, -5$ respectively (left to right).



*Mixture Model for tractography - Y Rathi et al.*

# ESTIMATING MIXTURE MODEL PARAMETERS

- Minimize the following cost function:

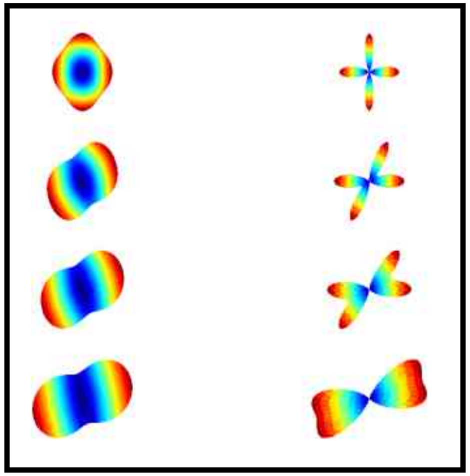
$$E(\mathbf{w},\mathbf{k},\mathbf{m}) = \| S(\mathbf{u}) - \sum_{i=1}^{m} w_i f(\mathbf{u} \mid k_i, \mathbf{m}_i) \|^2$$
$$- \gamma_1 \sum_{i=1}^{m} \log(w_i) + \gamma_2 \left(1 - \sum_{i=1}^{m} w_i\right)^2 ,$$

  The first term is the data fidelity term, second term ensures that the weights are positive and third term enforces that the weights sum to 1.

- E is minimized using the Levenberg-Marquardt nonlinear optimizer.

# FIBER-ODF FROM DIFFUSION-ODF

- Once the parameters are estimated, the fiber-ODF can be directly computed from the diffusion-ODF, i.e., by scaling the parameter k. The following shows dODF and the corresponding fODF for a particular k:



dODF fODF

*Mixture Model for tractography - Y Rathi et al.*

# TRACTOGRAPHY

- Estimated fODF can be used for probabilistic tractography.
- In this work, we propose to do deterministic tractography.
- Follow the principal diffusion direction (PDD) for each component.
- If angle between components is less than 25, we assume there is only one fiber and follow along the average direction.
- A $4^{th}$ order Runge-Kutta scheme is used to propagate the fiber.

# TRACTOGRAPHY….

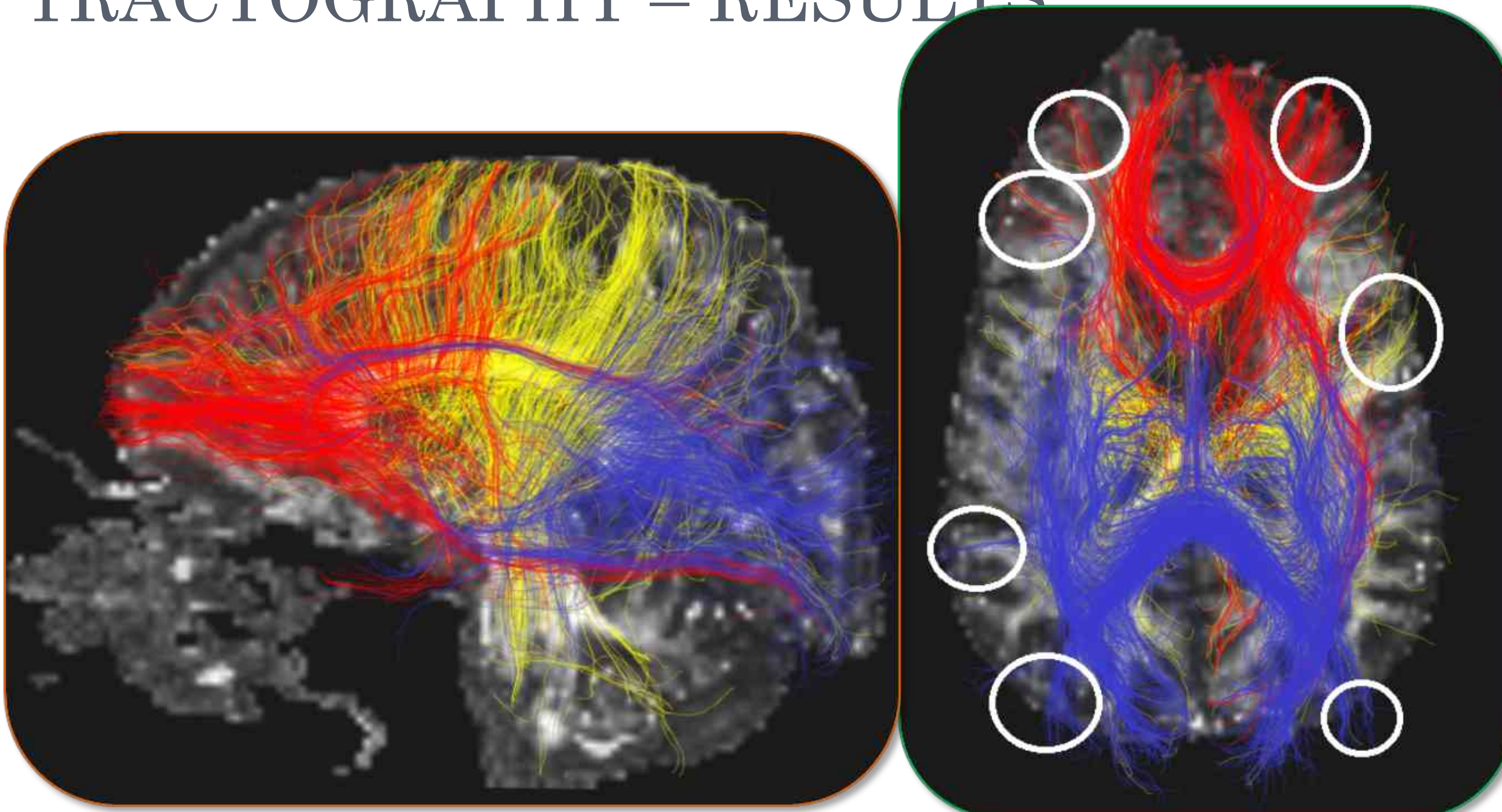
- Interpolation (cubic) is performed directly on the mixture model parameters at non-grid locations.

- In-vivo data: b-value = 1000, 51 gradient directions, 1.7 mm^3 spatial resolution.

- Performed tractography on the following regions:

1. Corpus Callosum (genu, splenium)
2. Corticospinal tract by seeding in the internal capsule

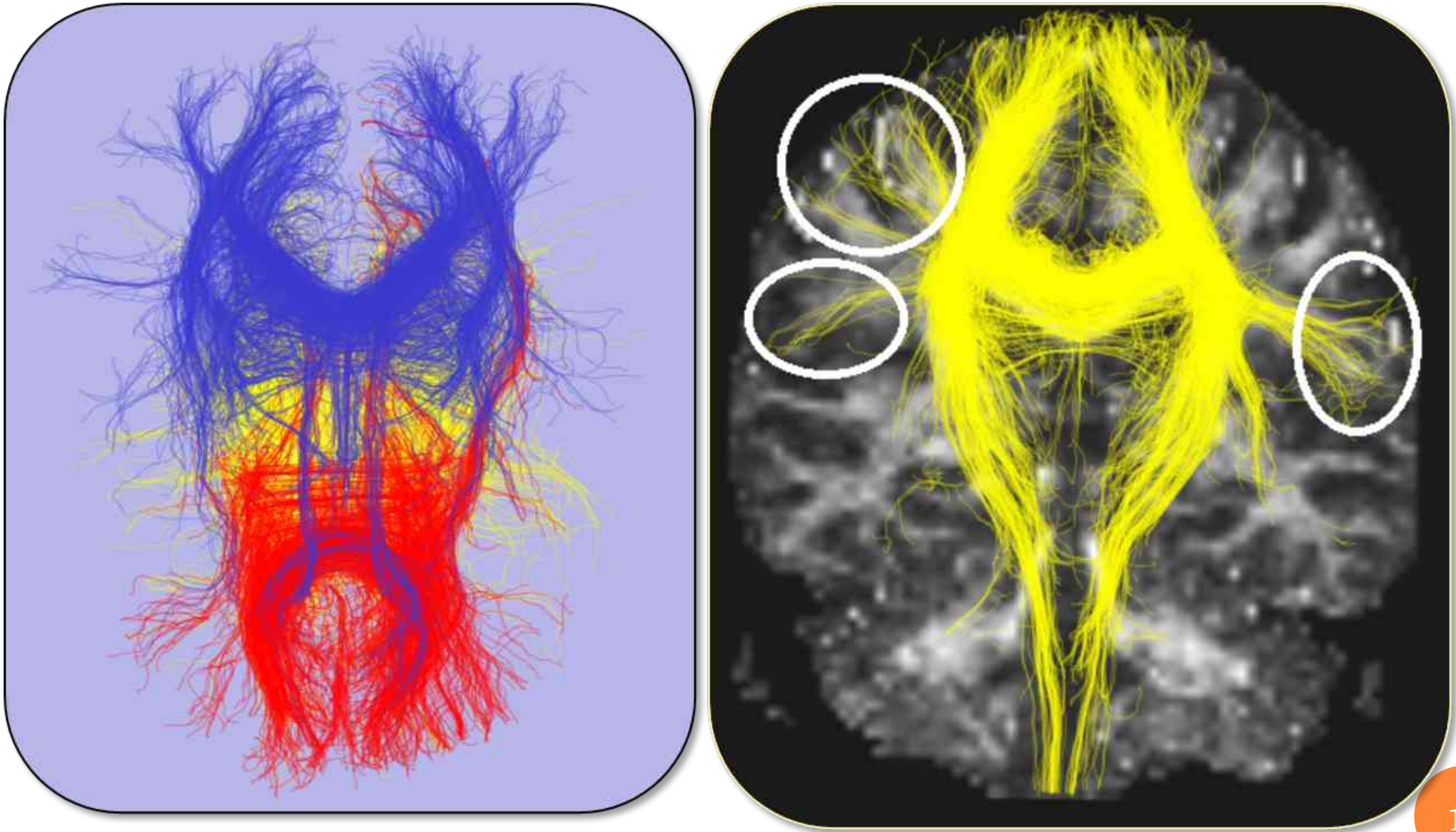
# TRACTOGRAPHY – RESULTS



Left: Sagittal view of corpus callosum, Right: axial view (white circles indicate fibers not detected using single-tensor tractography)

*Mixture Model for tractography - Y Rathi et al.*

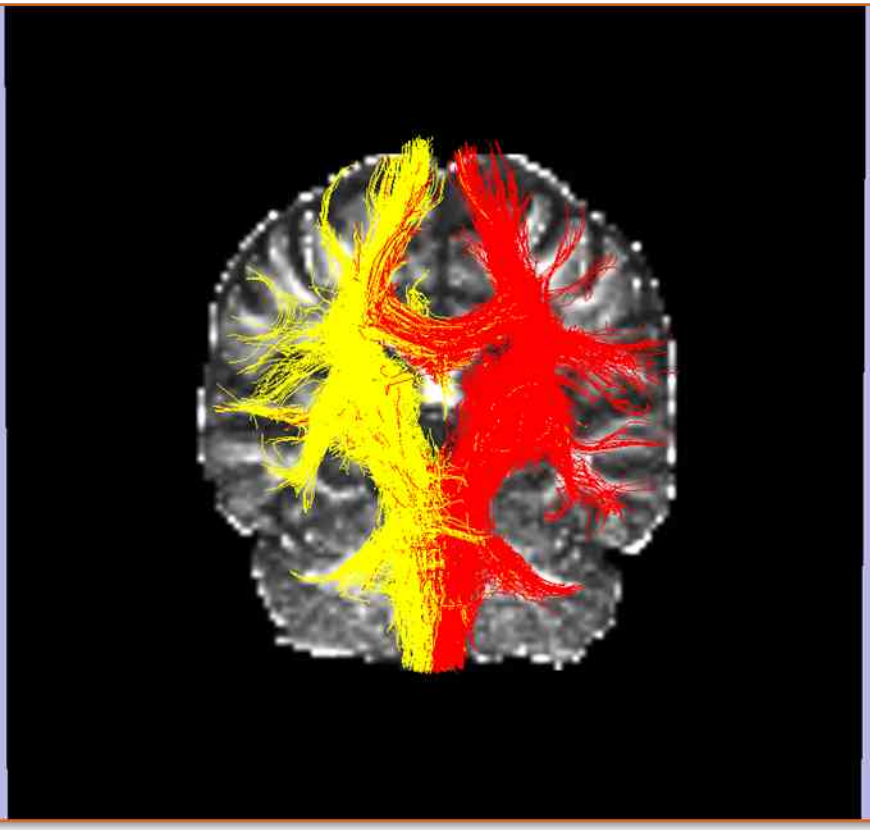
# TRACTOGRAPHY - RESULTS



Corpus callosum: Top view (left), coronal view (right) – white circles indicate fibers now detected using single-tensor tractography.

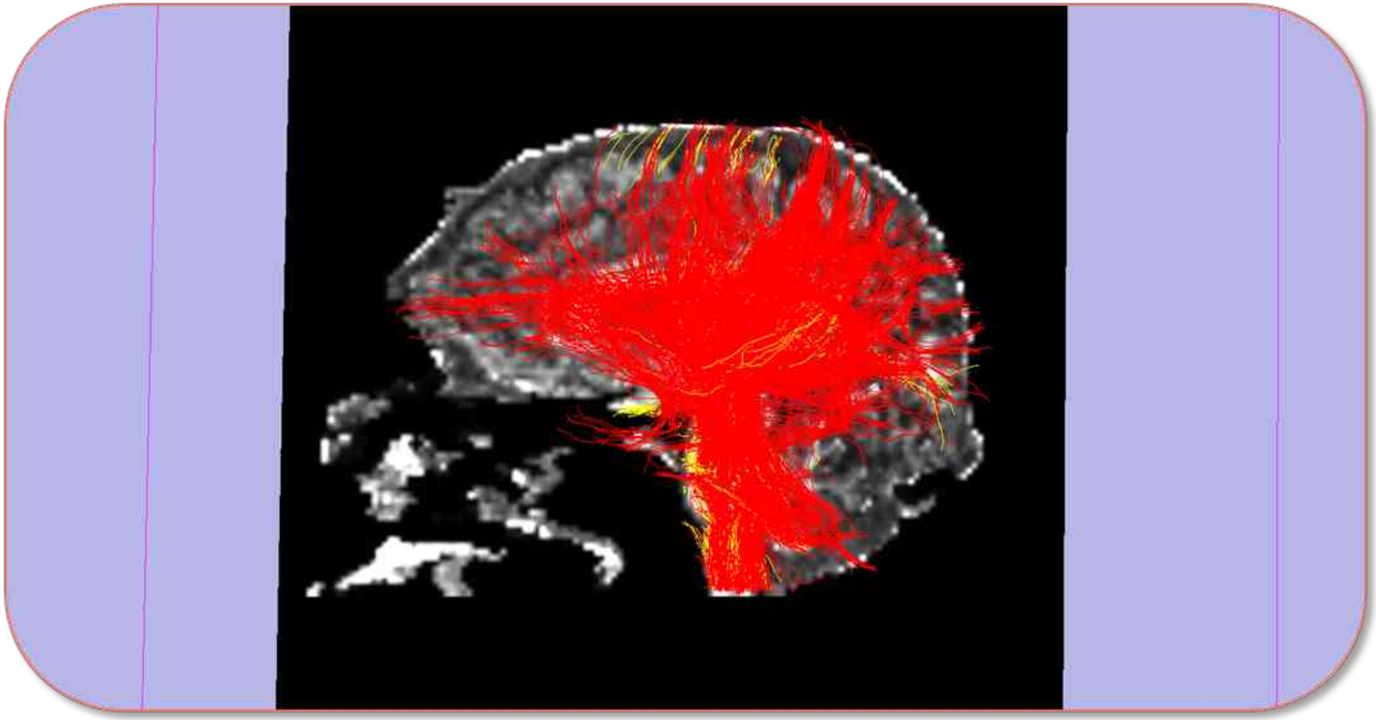
Mixture Model for tractography - Y Rathi et al.

# TRACTOGRAPHY RESULTS



Corticospinal tract (left and right hemisphere), seeding done in internal capsule.

# TRACTOGRAPHY RESULTS



Sagittal view of cortocospinal tract

# CONCLUSION

- Propose to use the Watson directional function for ODF estimation and tractography
- Analytical approximation of the fODF using the Funk-Radon transform
- Tractography finds fibers known to exist anatomically, yet undetectable using single-tensor tractography.
- However, fibers not connected are also picked-up due to partial voluming (although these can be removed in post-processing steps).