Mixture Model for estimating fiber ODF and multi-directional Tractography

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Introduction: Diffusion Tensor Imaging (DTI) is now a well-established scheme for analyzing neural pathways in the brain by means of streamline or probabilistic tractography. But DTI models the diffusion of water molecules by a Gaussian process, while the data captured by the diffusion-weighted MRI (DW-MRI) could very well be non-Gaussian. To overcome this limitation, a High Angular Resolution Diffusion Imaging (HARDI) scheme was proposed [1]. Several methods have been proposed to estimate the fiber orientation distribution function (fODF) from HARDI signals. Other methods estimate multiple tensors at each voxel and subsequently perform tractography [2]. In this work we propose to use a mixture of Watson orientation functions to model the fODF and subsequently perform multi-directional streamline tractography on real datasets.

Methods: Assuming diffusion in one principal direction, one can model the signal as follows: $S(u_i) = S_i = S_0 \exp\left(-k\left(\boldsymbol{u}_i^T \boldsymbol{m}\right)^2\right)$, where $\boldsymbol{u}_i$ is the ith gradient direction, $\boldsymbol{m}$ is the principal diffusion direction, k is a scaling parameter and S_0 is the signal without diffusion sensitizing. For diffusion in multiple directions, a mixture model can be used: $S_i = S_0 \sum_{j=1}^{N} w_j \exp\left(-k_j\left(\boldsymbol{u}_i^T \boldsymbol{m}_j\right)^2\right)$, where N is the number of components in the mixture and w_j gives the volume fraction of each component. In order to compute the diffusion ODF, the Funk-Radon transform [1] can be applied to S_i to obtain the following approximation for dODF : $dO_i = \sum_{j=1}^{N} w_j \exp\left(-0.5 k_j \sin^2(\theta_{ij})\right)$, where $\theta_{ij} = \cos^{-1}(u_i^T m_j)$. Further, the fiber ODF (fODF) can be directly obtained from dO by simply scaling the factor k_j of each component. Thus, if one estimates the parameters m_j and k_j of the mixture model, the computation of fODF is straightforward.

Estimating the Mixture Model: In order to estimate the parameters of the mixture model, we propose to minimize the following cost function: $E(k.m) = \| \boldsymbol{A} - \boldsymbol{S} \|^2 - \gamma \sum_{\{h=1\}}^{N} \log(k_h)$, where $\boldsymbol{A}$ is the signal obtained from the scanner with $A_i = A(\boldsymbol{u}_i)$ and **S** is the signal described by the mixture model discussed above. The first term in the above equation is the data fidelity term, ensuring a fit in the L^2 sense and the second term ensures that k stays positive. E can be minimized using the Levenberg-Marquardt optimizer. In this discussion, we have assumed equal weights for each of the components and the number of components N in the mixture was 2. Thus, for a mixture of N components, we require only 3N parameters to describe the ODF, i.e., 2N parameters for representing direction in spherical co-ordinates and N parameters for scale. This is in contrast to spherical harmonics of order l which requires $\frac{(l+1)(l+2)}{2}$ coefficients, i.e., for order 6 it requires 28 coefficients [3]. Thus, the proposed mixture model requires fewer parameters to represent the ODF. Further, the principal diffusion direction (PDD) is directly obtained as a consequence of estimating the model parameters, whereas for all non-parametric methods like spherical harmonics, a separate algorithm is required to extract the PDD. In particular, [3] uses a separate deconvolution operation to first obtain the fODF from the diffusion ODF and then extracts the PDD. In the proposed mixture model, fODF can simply be obtained by multiplying the scaling factor k by a constant, say 8, for a particular choice of the convolution kernel. Figure 1. shows the diffusion ODF and the corresponding fODF for various separation angles (90,60,45,30 degrees in that order).

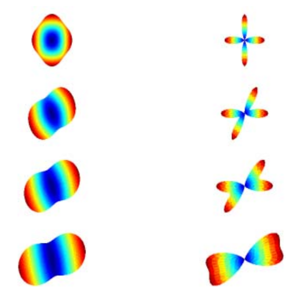

Figure 1. dODF and fODF.

Multi-Directional Streamline Tractography: The above formulation of fODF can be directly used for probabilistic and streamline tractography. In this work, we demonstrate streamline tractography assuming 2 components (PDD) per voxel. If, in reality, there is only one fiber at a voxel, the model estimation results in 2 PDD that are very close. If the angle between the 2 PDD is less than 25 degrees, we assume one fiber and propagate along the mean of the two angles. If the angle is greater than 25 degrees, we branch the fiber from that location. A fourth-order Runge-Kutta scheme was used for propagating the fiber. Note that, the parameters of the mixture model form a Euclidean vector space and hence interpolation can be easily performed at non-grid locations. Figure 2. shows the result of multi-directional streamline tractography for the corpus callosum. Seeding was done at each voxel inside the corpus callosum. The data was captured using a b-value of 1000 with 102 gradient directions on the sphere. Notice that it captures fibers that are known to exist anatomically but are missed by the single-tensor tractography algorithms.

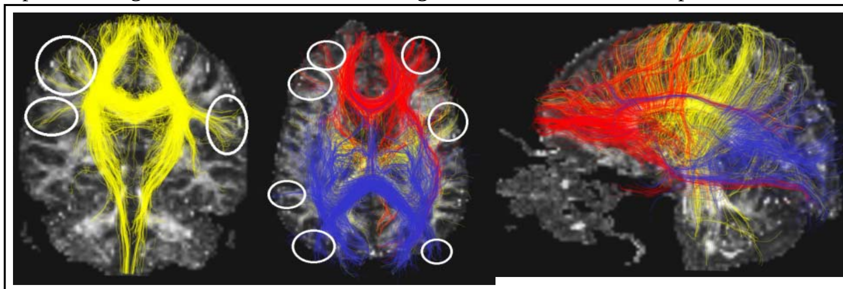

Figure 2. Coronal, Axial, Sagittal view (left tor right) of the corpus callosum fibers, with white circles indicating areas where one-tensor streamline tractography fails to find fibers.

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