SEEING THE UNSEEN: SEGMENTING WITH DISTRIBUTIONS

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ABSTRACT
An efficient method for separating an object from the background in an image is presented. The segmenting curve, corresponding to the object boundary, is represented as the zero level set of a signed distance function. Most existing region based methods in the geometric active contour framework perform segmentation by maximizing the separation of intensity moments inside and outside the evolving contour. We generalize these methods by minimizing the Bhattacharyya distance so that it separates regions of different distributions. Preliminary results show that the proposed method can segment low contrast, complex images with a very simple curve flow equation.

KEY WORDS
Geometric Active Contours, Bhattacharyya distance, level set method, partial differential equations.

1 Introduction

Image segmentation has been a topic of extensive research in the computer vision community; see [1, 2, 3, 4, 5, 6, 7] and the references therein. In particular, geometric active contours (GAC) have been successfully used for this task. Most of these methods use photometric information such as intensity, color or texture to segment an object. In the GAC framework, an image based energy functional, typically a function of the image intensity moments, is minimized to separate an object from the background. More specifically, a closed curve C, represented implicitly as the zero level set of a signed distance function [8, 9], is evolved so that it minimizes an image based energy functional. In this work, we propose to minimize a novel energy functional which represents the distance between the probability distribution function (pdf) inside and outside the curve C, i.e., the distance between the pdf of the object and the background, assuming that each region is realized by a unique random variable.

There is a large body of literature concerning the problem of separating an object from its background; see for example [1, 4] and the references therein. The level set framework has proven to be quite useful for this task. The authors in [10] use mean intensities to perform segmentation, which is a special case of the more general Mumford-Shah segmentation model [11]. In [12, 13, 14], the authors proposed a variational framework for segmenting an object using the first two moments (i.e., mean and variance) of image intensities. All the methods mentioned above utilize image statistics from the entire region, while there exist methods that use local information, i.e., edges [15, 16]. Most of the region based models have been inspired by the region competition technique of Zhu and Yuille [17]. Another method proposed by Freedman *et al.* [2], maximizes the Bhattacharyya distance between a known pdf and the pdf of the region inside C. This method requires "learning" the pdf of the desired region *a-priori*. Further, an additional approach that is very closely related to our work is given in [18]. Here, the authors propose an information theoretic approach to segment an object by maximizing the mutual information between the region labels and the image intensities. The curve evolution proceeds by computing the log-likelihood ratio of the points on the curve C. This method also employs information available from the pdf of the region inside and outside C.

In this paper, we propose to minimize the distance between pdf's by using the Bhattacharyya distance [19], in order to separate two regions with different pdf's without any *a-priori* knowledge about the object or background. The technique proposed in this present work is most closely related to the one in [18] which also uses pdf's to determine the optimal segmentation. However, our formulation of the problem and the resulting flow are quite different. Initial experimental results show that the performance of the proposed method is similar to that proposed by [18] and better than methods that use only the first two intensity moments. We should however note that, the proposed method uses a different metric (compared to [18]) to compute distance between two pdf's, and is simpler to understand and computationally less complex.

2 The Bhattacharyya Flow

An object can be represented by a closed curve enclosing its boundary. Many possible parameterizations of planar shapes described as closed contours have been proposed (see [20, 21] and the references therein). Recently, level set methods, which use an implicit representation of contours, have become very popular [8, 9]. The curve C is represented as the zero level set of a higher dimensional function, typically a signed distance function $\phi : \mathcal{R}^2 \rightarrow \mathcal{R}$, such that $\phi < 0$ inside C and $\phi > 0$ outside C. This representation allows for natural breaking and merging of curve topologies, hence we have decided to use it in the present work.

The Bhattacharyya distance [19] gives a measure of similarity between two pdf's, i.e.,

$$B = \int_{\mathcal{Z}} \sqrt{P_{in}(z) P_{out}(z)}\, dz, \tag{1}$$

where $z \in \mathcal{Z}$ is a photometric variable such as intensity, a color vector or a texture vector, and lives in the space $\mathcal{Z}$, while P_{in} and P_{out} are pdf's defined on the variable z for the inside and outside regions respectively. This measure varies between 0 and 1, where 0 indicates a complete mismatch and 1 indicates complete agreement between the pdf's. Let $x \in \mathcal{R}^2$ specify the coordinates in the image plane, and let $I : \Omega \subset \mathcal{R}^2 \rightarrow \mathcal{Z}$ be a mapping from the image plane to the space of the photometric variable. The pdf P_{in} (or P_{out}) is assumed to be defined by

$$P_{in}(z) = \frac{\int_\omega K(z - I(x))\, dx}{\int_\omega dx} = \frac{\int_\omega K(z - I(x))\, dx}{A_{in}} \tag{2}$$

which is the nonparametric kernel density estimate of the pdf of z for a given kernel K. Typical choices for K are the Dirac delta function $\delta(.)$ and the Gaussian kernel given by $K(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-y^2/2\sigma^2\right)$. The rest of the derivation is independent of the choice of the kernel K. For the case of curve evolution, P_{in} is the density of the region inside the curve C. Thus, ω is the region enclosed by C. Writing (2) in terms of the level set function ϕ, we get

$$P_{in}(z) = \frac{\int_\Omega K(z - I(x))\; H(-\phi(x))\, dx}{\int_\Omega H(-\phi(x))\, dx}, \tag{3}$$

where H is the Heaviside step function given by:

$$H(\phi) = \begin{cases} 1 & \phi > \epsilon\ , \\ 0 & \phi < -\epsilon\ , \\ \frac{1}{2}\{1 + \frac{\phi}{\epsilon} + \frac{1}{\pi}\sin\left(\frac{\pi\phi}{\epsilon}\right)\} & \text{else,} \end{cases}$$

and Ω is the whole image domain. Similarly, $P_{out}(z)$ can be written as

$$P_{out}(z) = \frac{\int_\Omega K(z - I(x))\; H(\phi(x))\, dx}{\int_\Omega H(\phi(x))\, dx}. \tag{4}$$

Computing the first variation of (1), we get the following:

$$\frac{\partial P_{in}(z)}{\partial \phi} = \frac{\delta_\epsilon(\phi)}{A_{in}}(P_{in}(z) - K(z - I(x)))\ ,$$

$$\frac{\partial P_{out}(z)}{\partial \phi} = \frac{\delta_\epsilon(\phi)}{A_{out}}(K(z - I(x)) - P_{out}(z))\ ,$$

$$\nabla_\phi B = \frac{1}{2}\int_{\mathcal{Z}} (P_{in}(z)\; P_{out}(z))^{-1/2} \times \left(\frac{\partial P_{in}(z)}{\partial \phi} P_{out}(z) + P_{in}(z)\frac{\partial P_{out}(z)}{\partial \phi}\right) dz.$$

Combining all of the equations above, we obtain the following PDE:

$$\frac{\partial \phi(x,t)}{\partial t} = -\frac{B\delta_\epsilon(\phi)}{2}\left(\frac{1}{A_{in}} - \frac{1}{A_{out}}\right) - \frac{\delta_\epsilon(\phi)}{2} \times \int_{\mathcal{Z}} K(z - I(x)) \left(\frac{1}{A_{out}}\sqrt{\frac{P_{in}(z)}{P_{out}(z)}} - \frac{1}{A_{in}}\sqrt{\frac{P_{out}(z)}{P_{in}(z)}}\right) dz, \tag{5}$$

where A_{in} and A_{out} is the area inside and outside the curve, respectively. The first term in this equation determines the "global" direction in which the entire curve moves, whereas the second term determines the "local" evolution direction. Thus, the initial motion of the curve is influenced by the "global" term, while its contribution is minimal when B is close to zero indicating convergence of the curve evolution.

It should be noted that the above evolution equation is quite general and can be used with vector valued variable z as in the case of color images or with the output of a filter bank which captures texture information. In the present work we restrict our experiments to the case where z is the set of gray level values in the set $\{1, 2, ..., 256\}$. A detailed analysis for other types of photometric information is the subject of future research. For numerical experiments in this work, we have used $K(z - I(x)) = \delta(z - I(x))$, with $I(x)$ being the gray level intensity values and $\delta(.)$ is a smooth Gaussian kernel with a predefined variance. A smooth approximation was used for $\delta_\epsilon(\phi)$, i.e.,

$$\delta_\epsilon(\phi) = \begin{cases} 0 & \phi > \epsilon,\ \phi < -\epsilon, \\ \frac{1}{2\epsilon}\left(1 + \cos(\frac{\pi\phi}{\epsilon})\right) & \text{otherwise.} \end{cases} \tag{6}$$

In numerical experiments, a regularizing term is added penalizing the curve length so that the contour is smooth, i.e.,

$$\int_\Omega \parallel \nabla H(\phi) \parallel \; dx.$$

Thus, the final expression for the level set evolution is given by

$$\frac{\partial \phi}{\partial t} = \delta_\epsilon(\phi)\,(V + \alpha\,\kappa) \tag{7}$$

where κ is the curvature, α is a user defined weight and V is the speed term from equation (5):

$$V = -\frac{B}{2}\left(\frac{1}{A_{in}} - \frac{1}{A_{out}}\right) - \frac{1}{2} \times \int_{\mathcal{Z}} K(z - I(x)) \left(\frac{1}{A_{out}}\sqrt{\frac{P_{in}(z)}{P_{out}(z)}} - \frac{1}{A_{in}}\sqrt{\frac{P_{out}(z)}{P_{in}(z)}}\right) dz.$$

3 Experiments

In this section, we demonstrate the segmentation results obtained by applying the above PDE on several images. In the first example, we segment the classic image of a zebra. This image has been segmented successfully by several other methods. In [22], a small patch of the texture of the zebra was learned *a-priori* and this knowledge was used in the segmentation process. The authors in [23] minimize the K-L divergence measure between two Laplacian distributions to obtain appropriate segmentation. The method in [24] uses a stochastic active contour method to lock on to the sharp edges of the zebra. This method, however, requires the edges to be relatively sharp, which makes it vulnerable to noisy environment. The proposed method however suffers from no such limitations as will become clear from the next set of examples. Figure 1 shows the initial contour and the final segmentation result. The segmentation is a natural consequence of separating the bimodal distribution of the zebra and unimodal distribution of the background. The method proposed in [18] can in principle segment such trimodal images as was demonstrated by segmenting an image of a leopard. Thus, the proposed method is an alternative to the one presented in [18]. We should note however that the method proposed in the current work follows directly by taking the first variation of the Bhattacharyya distance, whereas the one proposed in [18] requires an approximation to compute the entropy and taking the first variation of a complicated energy functional with double region integrals, which makes it less efficient from the computational point of view.

Medical images are inherently noisy and have poor contrast. The proposed method provides a natural way to segment such images. In our second example, we segment a slice of the caudate nucleus. The starting contour and the corresponding distributions P_{in} and P_{out} are shown in Figure 2. The final segmentation along with the densities is also shown. Figure 2 shows the segmentation result using the algorithm of [4] which uses the first two intensity moments. This example demonstrates the robustness of the proposed method in cases where the first two intensity moments fail to give the desired results.

The third example demonstrates the power of the proposed method to segment regions with same mean and variance. This toy example was generated by adding Gaussian noise to the original image to create regions with the required distributions. Figure 3 shows the progression of the segmenting contour along with the final distributions P_{in} and P_{out}. This example shows that the proposed method can segment objects layered on a textured background.

Finally, a toy example was contrived to demonstrate the power of the proposed method : the ability to separate regions indistinguishable by the human eye. The background in this synthetic image was generated by sampling from a Rayleigh distribution : $x \sim p(x) = xe^{\frac{-x^2}{2}}$ with mean $m = \sqrt{\frac{\pi}{2}}$. The object (referred below as cat) was generated by sampling from a different random variable y whose distribution is related to x via $y = 2m - x$. It can be easily shown that, the object and the background (x and y) have the same mean and variance. Figure 4 shows the original image and the generated image. Segmenting such

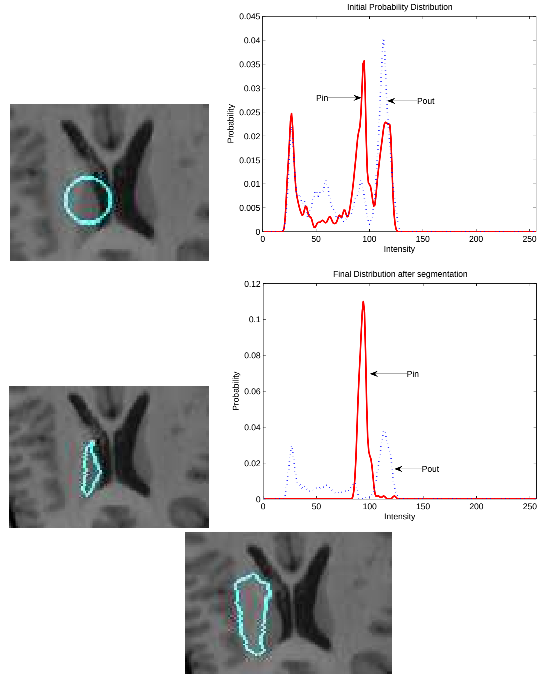

Figure 2. Caudate: Initial and final segmentation with the corresponding densities. Dark line gives the inside density P_{in} while dotted line shows P_{out}. The bottom segmentation uses the method in [4] with the first two intensity moments, which yields an incorrect segmentation.

an image is indeed a challenge. Figure 5 and 6 show the different stages of the evolving contour, along with the initial, final and actual distribution inside and outside the cat. As is clear, the final distribution is very close to the actual pdf's.

4 Conclusion and Future Research

In this work, we have proposed a novel method to segment an image in the geometric active contour framework by minimizing the Bhattacharyya distance between the pdf inside and outside the evolving contour C. The method is capable of separating regions even with the same mean and variance but differing only in the third and higher order moments. However, a detailed comparison of the proposed method with that in [18] is the subject of future research. In particular, we should note the work in [25], where the authors have shown that the Bhattacharyya distance performs better than the K-L divergence or Fisher ratio for a set of analytically known pdf's. We are in the process of comparing and contrasting these methods in the present framework in terms of their segmentation ability and rate of convergence.

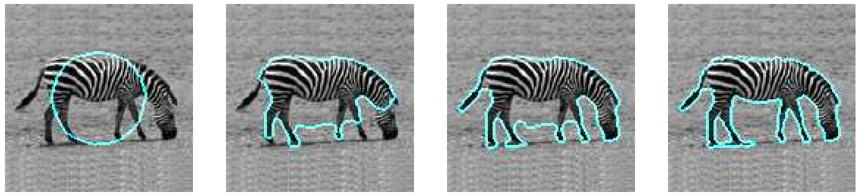

Figure 1. Segmenting the zebra: initial, intermediate and final contour

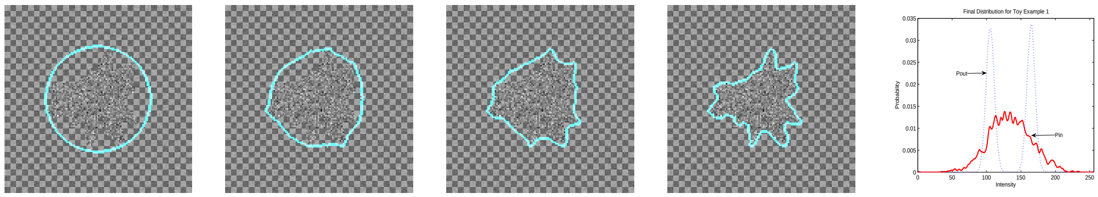

Figure 3. Toy Example 1: Same mean and variance. Last figure gives the distribution for the final segmentation.

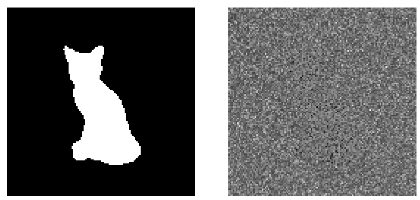

Figure 4. Toy Example 2:Original image (left) and Generated noisy image (right).

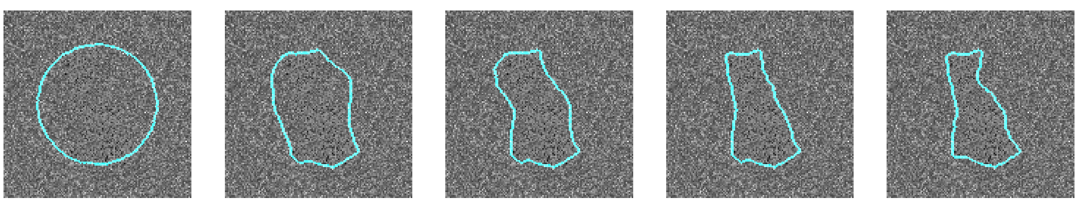

Figure 5. Toy Example 2: Same mean and variance inside and outside the object. Initial, intermediate and final contours.

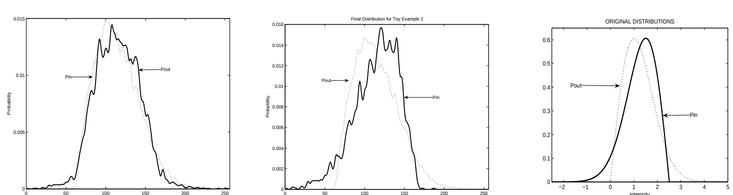

Figure 6. Toy Example 2:(From left to right) Distribution for the starting segmentation, final segmentation and actual distribution.

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