Graph cut segmentation with nonlinear shape priors

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Motivation

Challenges of graph cut segmentation

- Objects may have weak edges
- Surrounding clutter similar to object
- Occlusion

Existing shape-based approaches

- Limited shape variation
- Single fixed shape prone to misalignment
- Computationally intensive

Our contribution

- Incorporation of highly variable nonlinear shape priors into existing iterative graph cut methods

Graph cut segmentation

Overview

- Efficient global energy minimization:

$$E(A) = \sum_{p \in \mathcal{I}} R_p(a_p) + \lambda \sum_{(p,q) \in \mathcal{N}} B_{(p,q)},$$

where $A = \{a_p : a \in \{\mathcal{O}, \mathcal{B}\}, p \in \mathcal{I}\}$.

- Regional data term $R(a_p)$ and boundary smoothness term $B_{(p,q)}$
- Typically, region term taken to be the negative log-likelihood of a pixel's fit into the histogram:

$$R_p(\mathcal{O}) = -\ln P(\mathcal{I}_p|\mathcal{O}) \qquad R_p(\mathcal{B}) = -\ln P(\mathcal{I}_p|\mathcal{B})$$

yet this assumes a uniform prior.

- Often produces undesired segmentations
  - May not capture weak edges
  - May leak out of object of interest
  - Unable to capture occluded regions

Kernel PCA

- Form statistical model of training set
- Model captures modes of variation via principle component analysis (PCA)
- Use nonlinear kernel function for inner product distances when determining modes of variation:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(\frac{-\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}\right)$$

- For arbitrary $\mathbf{x}$, the pre-image $\hat{\mathbf{x}}$ is the closest point to $\mathbf{x}$ respecting the model. Can be approximated as a linear combination of training shapes weighted by distance:

$$\hat{\mathbf{x}} = \frac{\sum d(\mathbf{x}, \mathbf{x}_i)\mathbf{x}_i}{\sum d(\mathbf{x}, \mathbf{x}_i)}$$

Proposed algorithm

Overview

- Refine both intensity priors and shape priors through iterative application of graph cut
- First iteration forms priors from user initialization
- Subsequent iterations use the previous segmentation to calculate priors

New regional terms

- Non-uniform priors formed from pre-image

$$P(\mathcal{O}) = \hat{\mathbf{x}}$$
$$P(\mathcal{B}) = 1 - P(\mathcal{O})$$

- Priors incorporated into regional term in Bayesian manner:

$$\begin{aligned} R_p(\mathcal{O}) &= -\ln(P(\mathcal{I}_p|\mathcal{O})P(\mathcal{O}_p)) \\ &= -\ln P(\mathcal{I}_p|\mathcal{O}) - \mu \ \ln P(\mathcal{O}_p) \\ R_p(\mathcal{B}) &= -\ln P(\mathcal{I}_p|\mathcal{B}) - \mu \ \ln P(\mathcal{B}_p) \end{aligned}$$

Algorithm

1. Compute histograms for intensity priors $P(\mathcal{I}_p|\mathcal{O})$ and $P(\mathcal{I}_p|\mathcal{B})$.
2. Compute pre-image $\hat{\mathbf{x}}$ and form shape priors $P(\mathcal{O})$ and $P(\mathcal{B})$.
3. Calculate edge weights $R(a_p)$ and $B_{(p,q)}$
4. Graph cut segmentation
5. Repeat until convergence

Results

User initialization, segmentation without shape, segmentation with proposed shape *(left to right)*: