### Challenges of graph cut segmentation
- Objects may have weak edges
- Surrounding clutter similar to object
- Occlusion

### Existing shape-based approaches
- Limited shape variation
- Single fixed shape prone to misalignment
- Computationally intensive

### Our contribution
- Incorporation of highly variable nonlinear shape priors into existing iterative graph cut methods

### Overview

#### Efficient global energy minimization:
\[
E(A) = \sum_{p \in \mathcal{P}} R_p(a_p) + \lambda \sum_{(p,q) \in \mathcal{N}} B_{(p,q)}
\]
where \( A = \{ a_p : a \in \{0, 1\}, p \in \mathcal{P}\} \).
- Regional data term \( R(a_p) \) and boundary smoothness term \( B_{(p,q)} \)
- Typically, region term taken to be the negative log-likelihood of a pixel's fit into the histogram:
  \[
  R_p(O) = -\ln P(\mathcal{O}_p) \quad R_p(B) = -\ln P(\mathcal{B}_p)
  \]
yet this assumes a uniform prior.
- Often produces undesired segmentations
  - May not capture weak edges
  - May leak out of object of interest
  - Unable to capture occluded regions

#### Existing iterative graph cut methods
yet this assumes a uniform prior.

### Kernel PCA
- Form statistical model of training set
- Model captures modes of variation via principle component analysis (PCA)
- Use nonlinear kernel function for inner product distances when determining modes of variation:
  \[
  k(x_i, x_j) = \exp \left( -\frac{\|x_i - x_j\|^2}{2\sigma^2} \right)
  \]
- For arbitrary \( x \), the pre-image \( \hat{x} \) is the closest point to \( x \) respecting the model. Can be approximated as a linear combination of training shapes weighted by distance:
  \[
  \hat{x} = \sum \frac{d(x_i, x_j)}{\sum d(x_i, x_j)}
  \]

#### New regional terms
- Non-uniform priors formed from pre-image
  - Priors incorporated into regional term in Bayesian manner:
    \[
    R_p(O) = -\ln P(\mathcal{O}_p|O_p) - \mu \ln P(\mathcal{O}_p) \\
    R_p(B) = -\ln P(\mathcal{B}_p|B_p) - \mu \ln P(\mathcal{B}_p)
    \]

### Proposed algorithm
1. Compute histograms for intensity priors \( P(\mathcal{O}_p) \) and \( P(\mathcal{B}_p) \).
2. Compute pre-image \( \hat{x} \) and form shape priors \( P(\mathcal{O}) \) and \( P(\mathcal{B}) \).
3. Calculate edge weights \( R(a_p) \) and \( B_{(p,q)} \).
4. Graph cut segmentation
5. Repeat until convergence

### Results
User initialization, segmentation without shape, segmentation with proposed shape (left to right):