Filtered Tractography
Validation on a Physical Phantom

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Diffusion Modeling and Fiber Cup
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method:
the model

multi-tensor mixture model

$$S(\boldsymbol{u}) = s_0 \sum_j w_j e^{-b \boldsymbol{u}^T D_j \boldsymbol{u}}$$

D_j	diffusion tensor
$\boldsymbol{u}$	unit direction
w_j	convex weights
b	acquisition constant
s_0	null signal (b=0)

model assumptions

...in this project

Two fibers

Fixed volume fractions

Tensors are elliptic or isotropic

model parameters

for two fibers...

...two principal directions $\boldsymbol{m} \in \mathbb{R}^3$

...two primary eigenvalues $\lambda_1 \in \mathbb{R}$

...two minor eigenvalues $\lambda_2 \in \mathbb{R}$

5 + 5 = 10 parameters

model parameters

for two fibers...

...two principal directions $\boldsymbol{m} \in \mathbb{R}^3$

...two primary eigenvalues $\lambda_1 \in \mathbb{R}$

...two minor eigenvalues $\lambda_2 \in \mathbb{R}$

5 + 5 = 10 parameters

$$S(\boldsymbol{u}) = 0.5\, s_0\, e^{-b\,\boldsymbol{u}^T D_{١}\boldsymbol{u}} + 0.5\, s_0\, e^{-b\,\boldsymbol{u}^T D_{٢}\boldsymbol{u}}$$

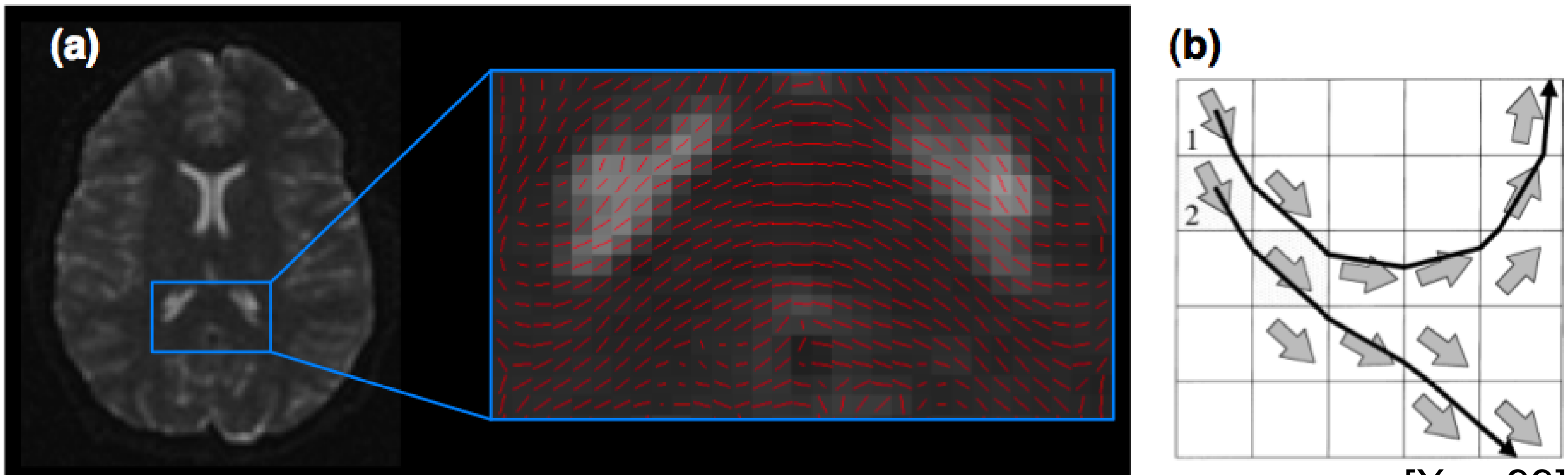
$$D_1 = \lambda_{11}\, \boldsymbol{m}_1 \boldsymbol{m}_1^T + \lambda_{21} (\boldsymbol{p}\,\boldsymbol{p}^T + \boldsymbol{q}\,\boldsymbol{q}^T)$$

eigenvectors: $\boldsymbol{m}$, $\boldsymbol{p}$, $\boldsymbol{q}$

method:
estimating the model

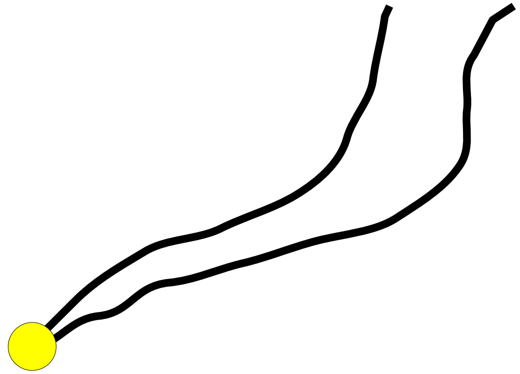
IPMI 2009
MICCAI 2009

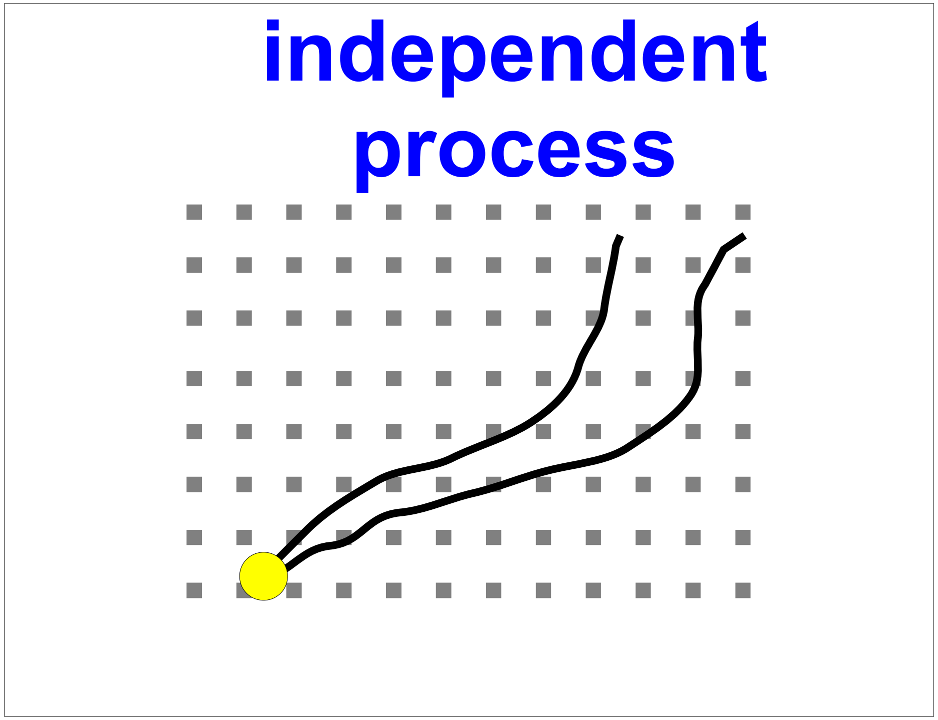
independent estimation

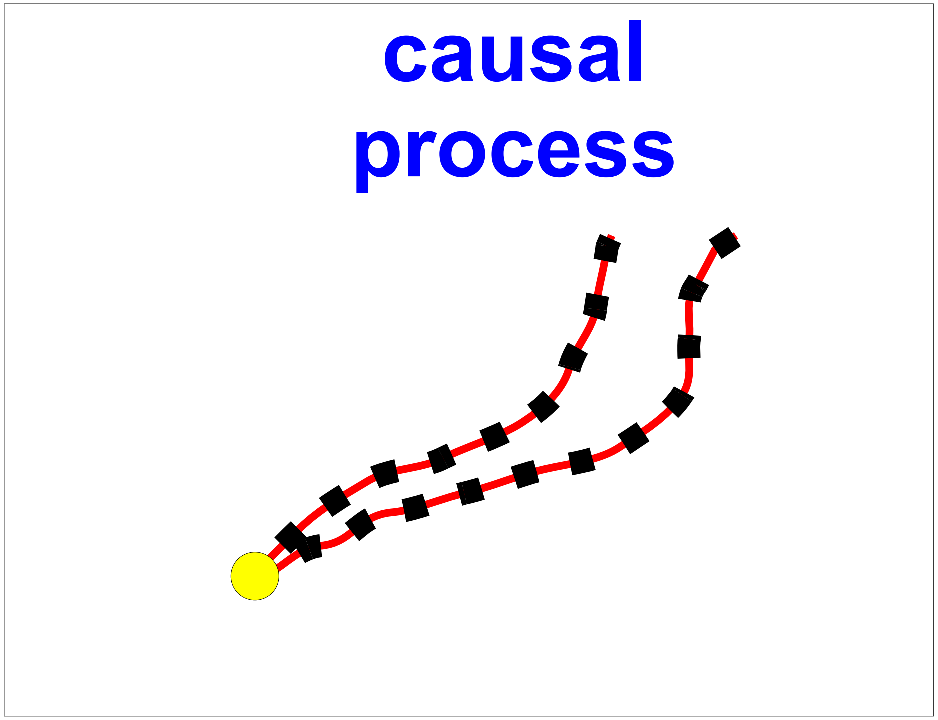


[Xue 99]

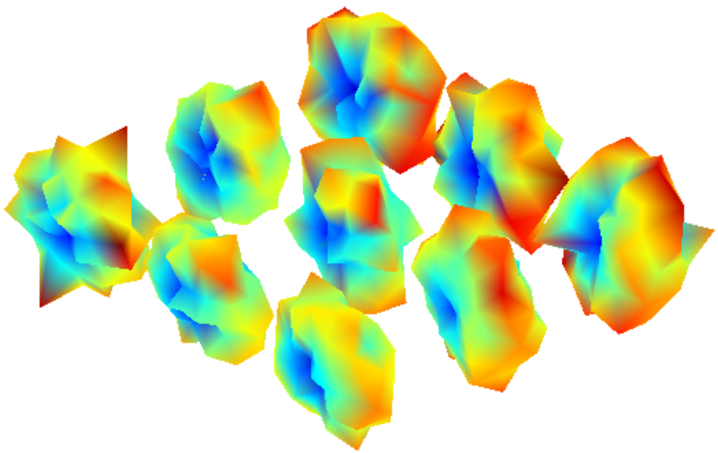
the system: a fiber

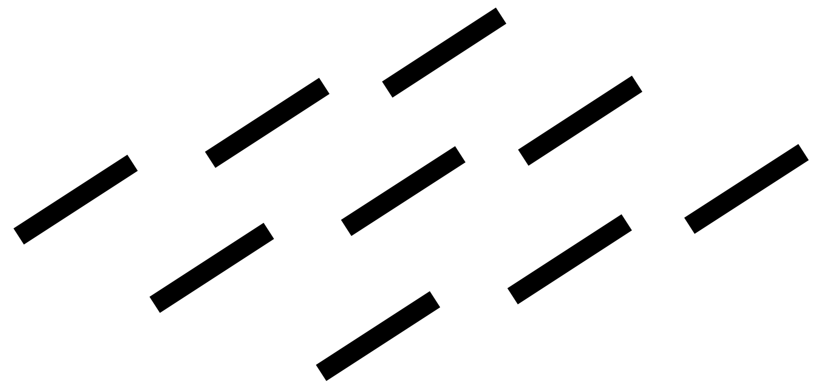






model-based filtering





objectives:

- estimate model from measurements
- suppress noise

notation

$\boldsymbol{x}_t$ state of system at time *t*

state = “model parameters”

$\boldsymbol{y}_t$ what you see at time *t*

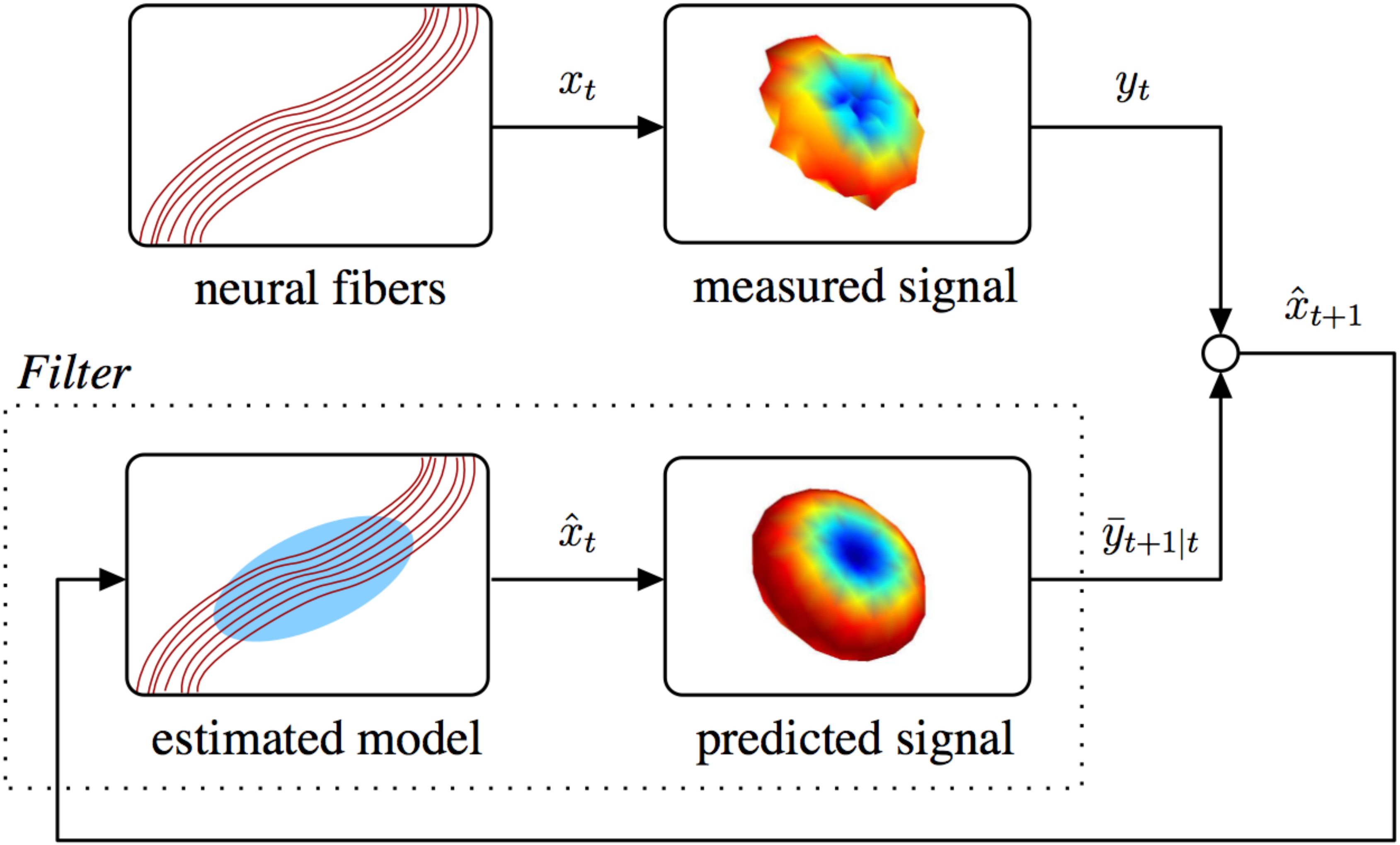
observation, measurement

update: $\boldsymbol{x}_{t+1} = F\,\boldsymbol{x}_t$ $\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t)$

observation: $\boldsymbol{y}_t = G\,\boldsymbol{x}_t$ $\boldsymbol{y}_t = g(\boldsymbol{x}_t)$

linear nonlinear

Kalman filtering



predict … measure … reconcile … repeat ...

$$\boldsymbol{x} = [\boldsymbol{m}_1 \lambda_{11} \lambda_{12} \boldsymbol{m}_2 \lambda_{21} \lambda_{22}]^T \in R^{10}$$

10 dimensional state

$\boldsymbol{y} \in R^m$ signal

$$\boldsymbol{x} = [\boldsymbol{m}_1 \lambda_{11} \lambda_{12} \boldsymbol{m}_2 \lambda_{21} \lambda_{22}]^T \in R^{10}$$

10 dimensional state

$$\boldsymbol{y} \in R^m$$ signal

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t) = \boldsymbol{x}_t$$

small steps slowly varying state

$$\boldsymbol{y}_t = g(\boldsymbol{x}_t) = S(\boldsymbol{u})$$

$$\boldsymbol{x} = [\boldsymbol{m}_1 \lambda_{11} \lambda_{12} \boldsymbol{m}_2 \lambda_{21} \lambda_{22}]^T \in R^{10}$$

10 dimensional state

$$\boldsymbol{y} \in R^m$$ signal

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t) = \boldsymbol{x}_t$$

small steps slowly varying state

$$\boldsymbol{y}_t = g(\boldsymbol{x}_t) = S(\boldsymbol{u})$$

$$y(\boldsymbol{u}) = S(\boldsymbol{u}) = 0.5\, s_0 e^{-b\boldsymbol{u}^T D_1 \boldsymbol{u}} + 0.5\, s_0 e^{-b\boldsymbol{u}^T D_2 \boldsymbol{u}}$$

$$D = \lambda_1 \boldsymbol{m}\boldsymbol{m}^T + \lambda_2 (\boldsymbol{p}\boldsymbol{p}^T + \boldsymbol{q}\boldsymbol{q}^T)$$

signal reconstruction is nonlinear

independent optimization

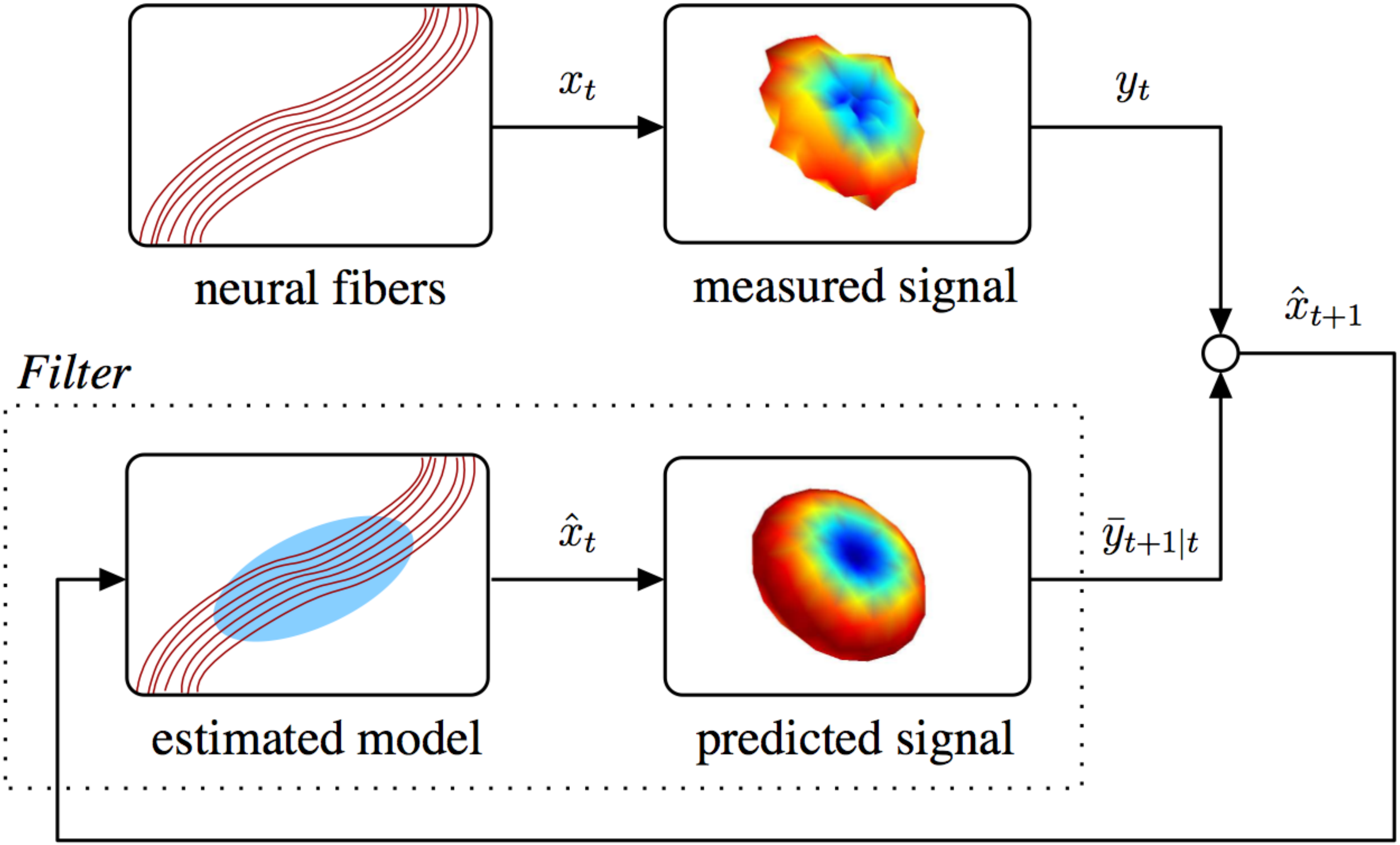
- least squares
  linearization
- gradient descent
  local minima
- Levenberg-Marquardt
  local minima

causal estimation

- extended Kalman filter
  mean + covariance
  linearization
- particle filter
  non-parametric
  sampling
- unscented Kalman filter
  mean + covariance
  no linearization
  limited sampling

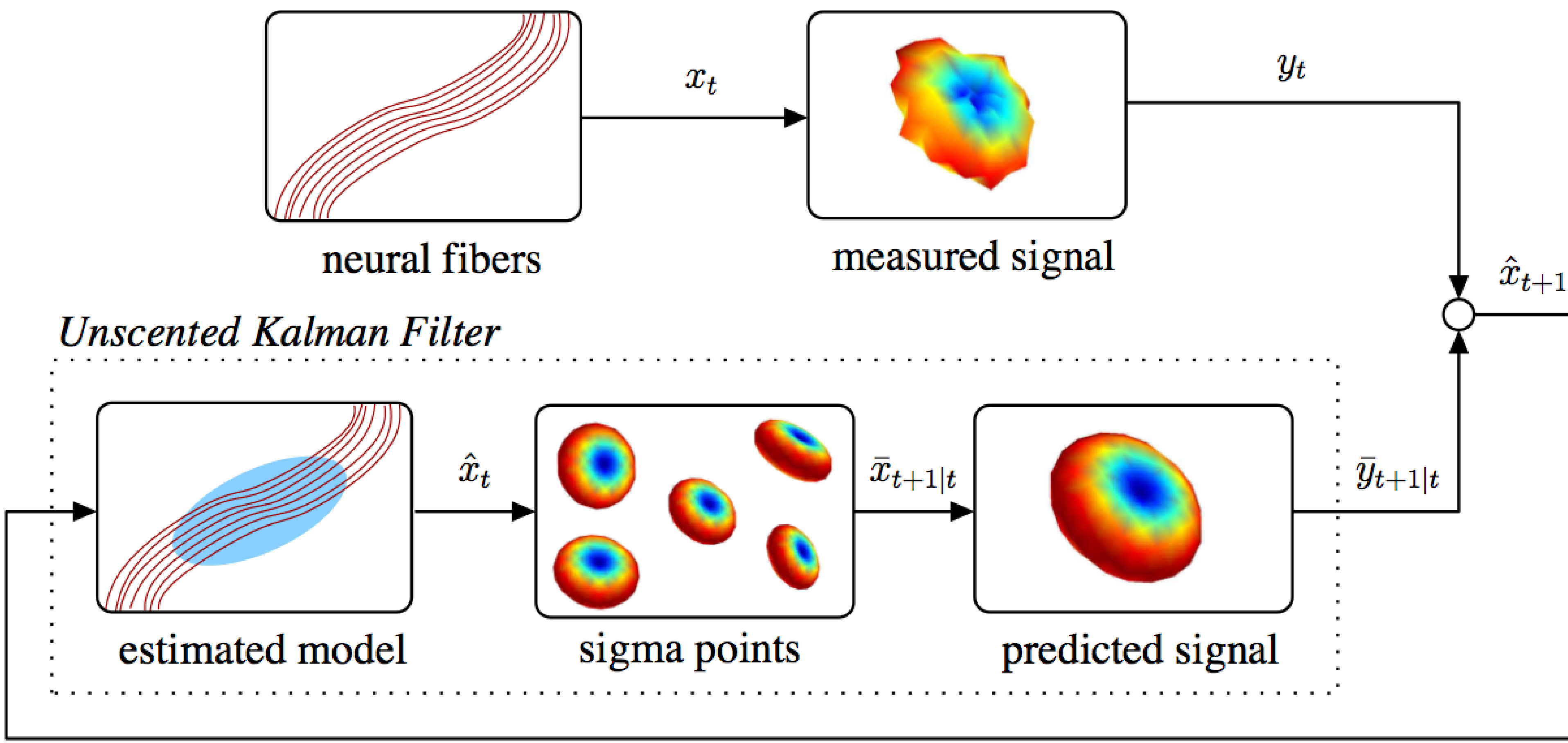
[Julier 04, Merwe 03]

linear Kalman filter



predict … measure … reconcile … repeat ...

unscented Kalman filter

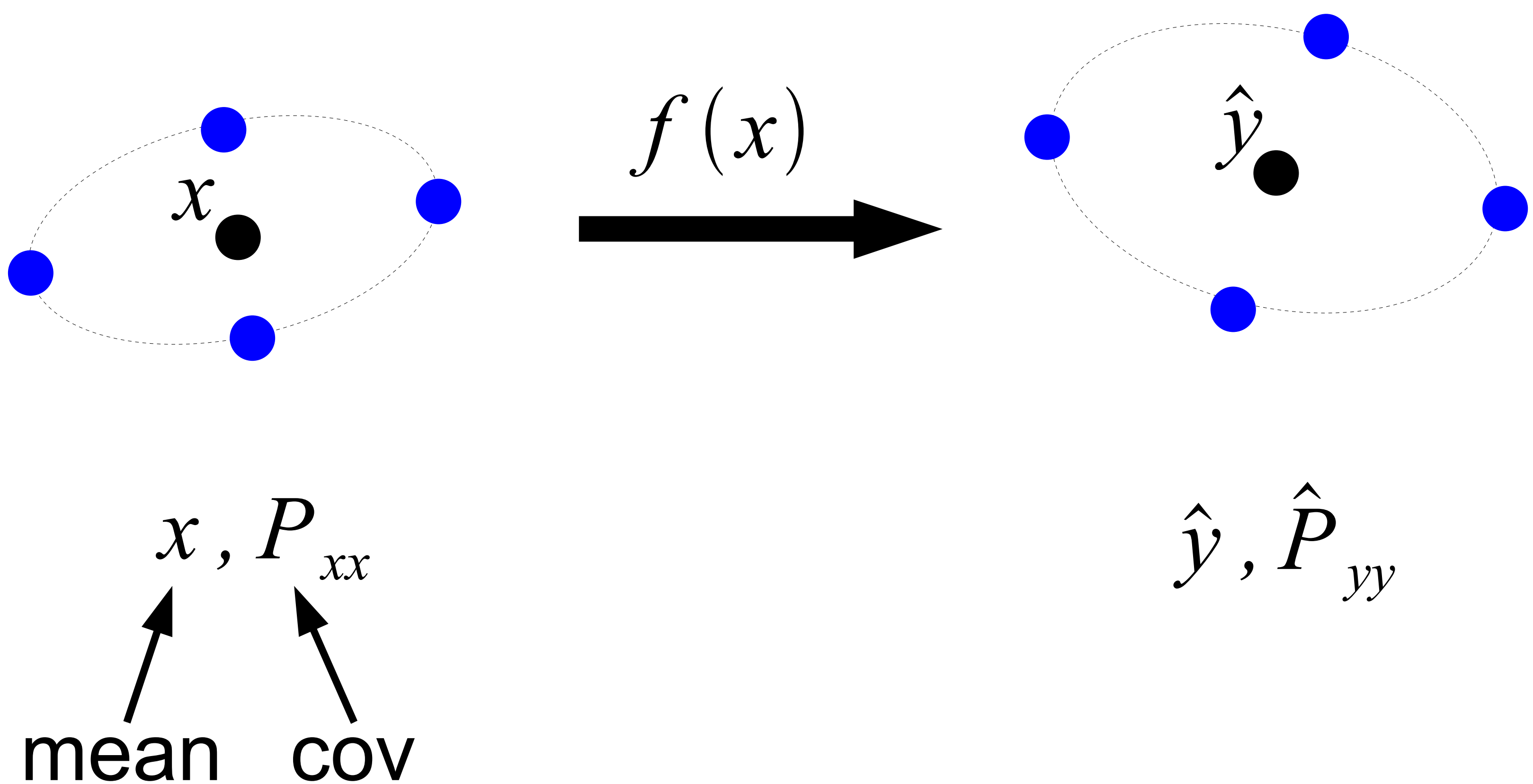


same update equations

modified prediction step

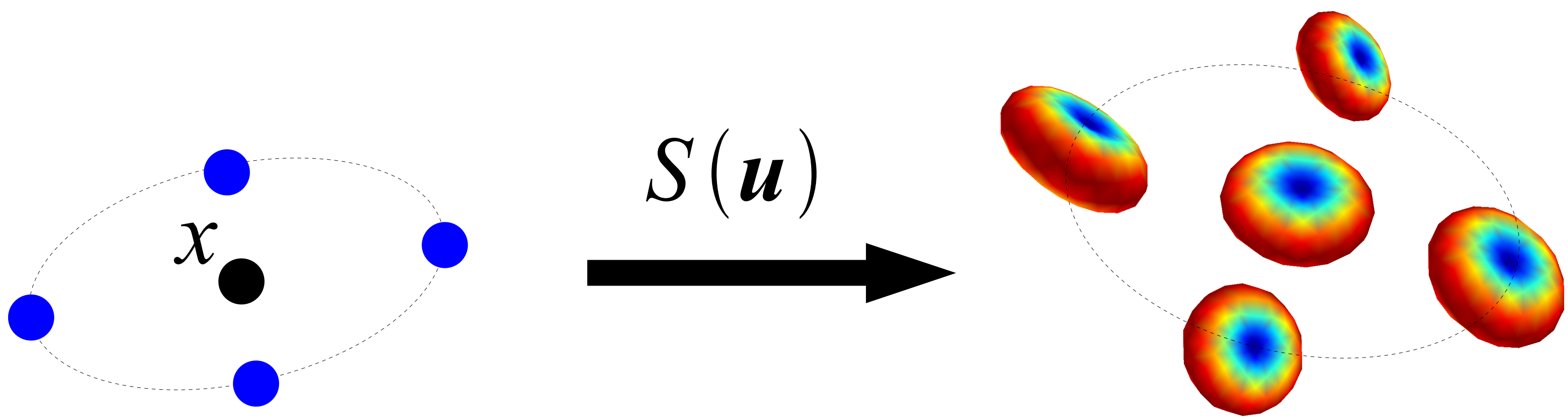
unscented transform

approximate the statistics...not the function



unscented transform

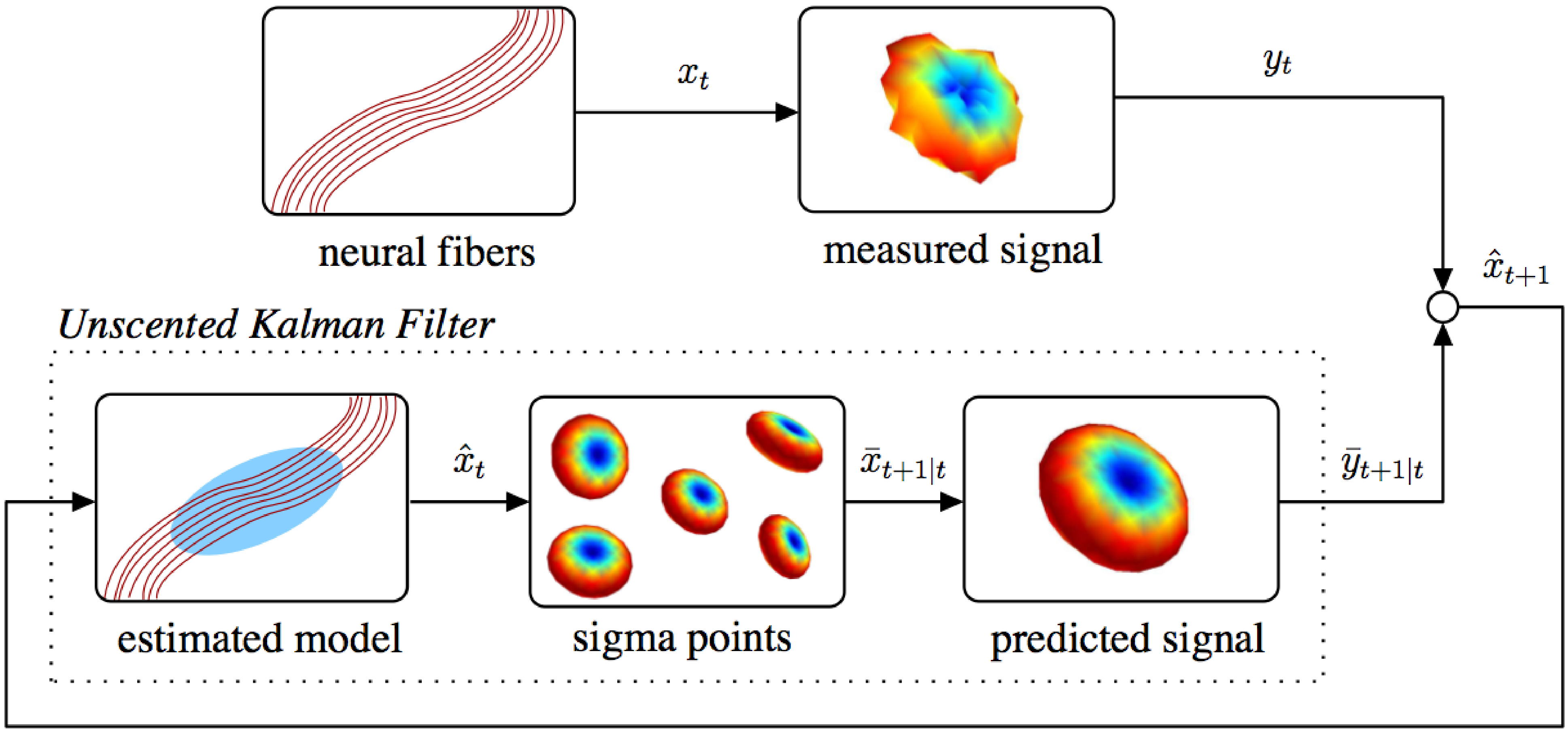
for signal reconstruction...



x, P_{xx} $\qquad\qquad$ $\hat{y}, \hat{P}_{yy}$

$$x = [\, \boldsymbol{m}_1 \lambda_{11} \lambda_{12} \boldsymbol{m}_2 \lambda_{21} \lambda_{22} \,]^T$$

unscented Kalman filter



predict … measure … reconcile … repeat ...

algorithm

interpolate scanner signal (measurement)

estimate model parameters with UKF

proceed in most consistent direction

terminate: FA < 0.15

the phantom

1)Seed throughout the mask (“full brain”)

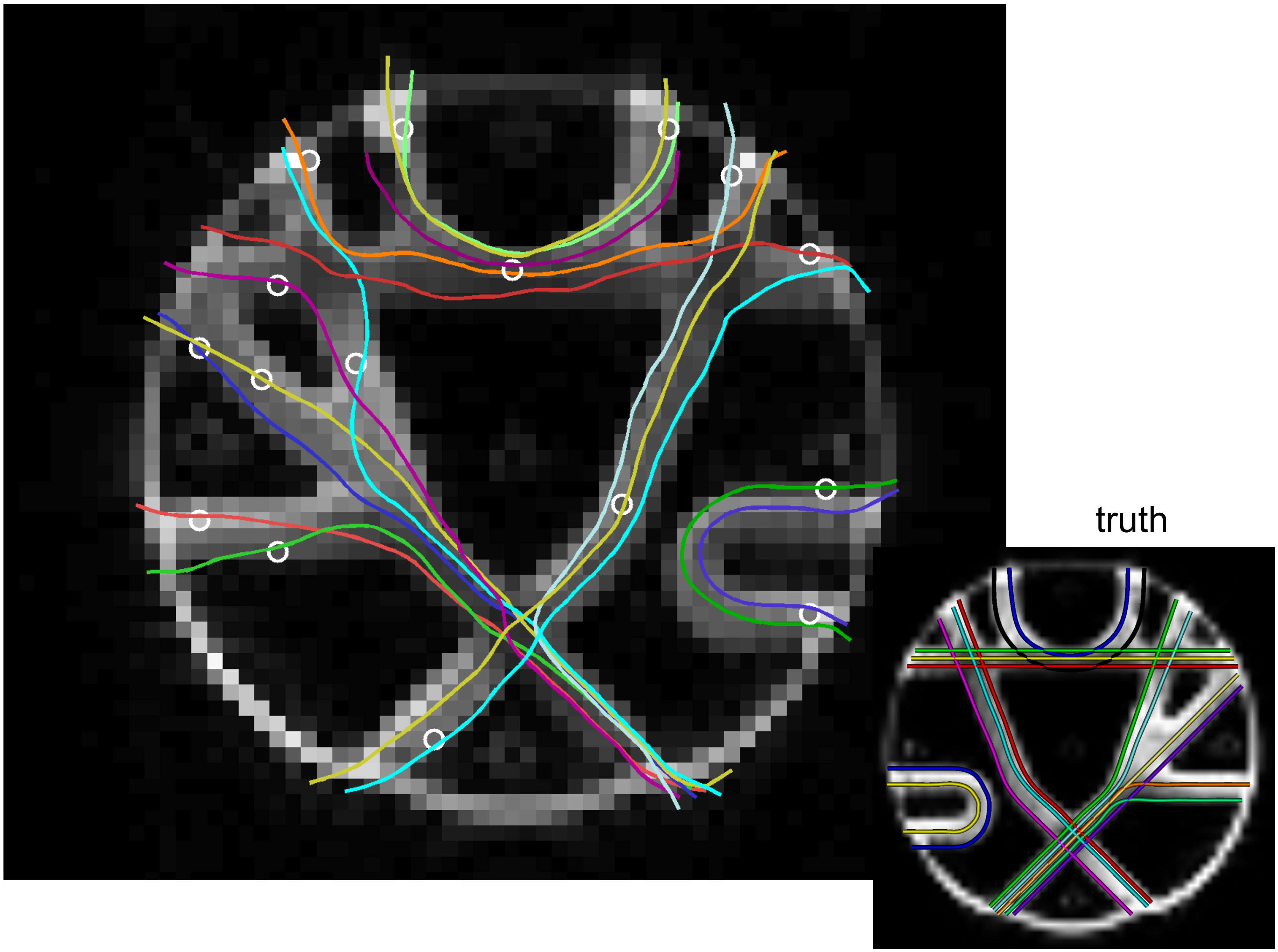
2)Select fibers passing through seed points

3)Manually select representative fiber

b=1500, 3mm

the phantom

3mm
b=1500



conclusion

inherent coherence along the fiber

we should exploit it in the estimation

Connectivity Studies

- Discrete paths
  - Pros: fast, tract-based studies
  - Cons: easily go off-track
- Probabilistic connectivity
  - Pros: handle uncertainty
  - Cons: leaking, difficult to interpret
- Filtered tractography
  - Accurate local estimates (mean, cov)
  - Probabilistic tractography

Local vs. Global

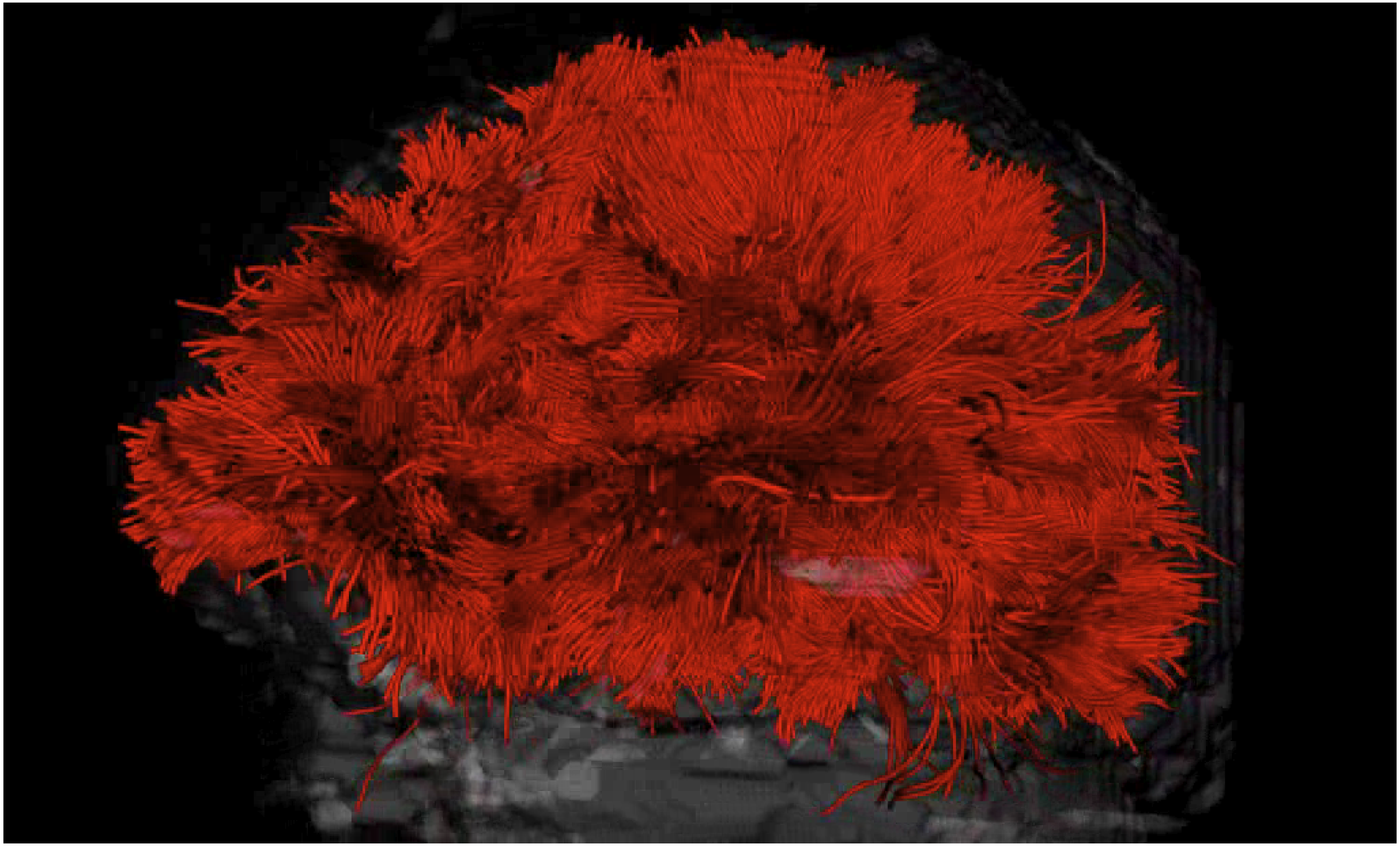
- Local methods easy go off track
- Global methods often over-regularize path

- Anatomic priors

- Hybrid
  - local: signal-model
  - global: path

- Filtered tractography
  - Replace local streamline in sampling
  - Covariance uncertainty indicates failure

end



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