Filtered Tractography:
State estimation in a constrained subspace

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Overview

Tractography is typically performed after estimating the principal diffusion directions in each voxel.

Problem:
Looking at each voxel independently, it is difficult to determine if a slight bump in the signal is a second component or simply noise, so many pathways remain undetected.

Solution:
Use a stable filtering framework to estimate the full diffusion model and use this to drive tractography.

Result:
This significantly improves the angular resolution at crossings and branchings while inherently adapting to noise.

Method

Summary: Existing techniques estimate the local fiber orientation at each voxel independently so there is no running knowledge of confidence in the estimated fiber model. We formulate fiber tracking as recursive estimation: at each step of tracing the fiber, the current estimate is guided by the previous.

Model: weighted mixture of two Gaussian tensors

$$S(\boldsymbol{u}) = w_1 s_0 e^{-b\boldsymbol{u}^T D_1 \boldsymbol{u}} + w_2 s_0 e^{-b\boldsymbol{u}^T D_2 \boldsymbol{u}} \qquad D_1 = \lambda_{11} \boldsymbol{m}_1 \boldsymbol{m}_1^T + \lambda_{21}(\boldsymbol{p}\boldsymbol{p}^T + \boldsymbol{q}\boldsymbol{q}^T)$$

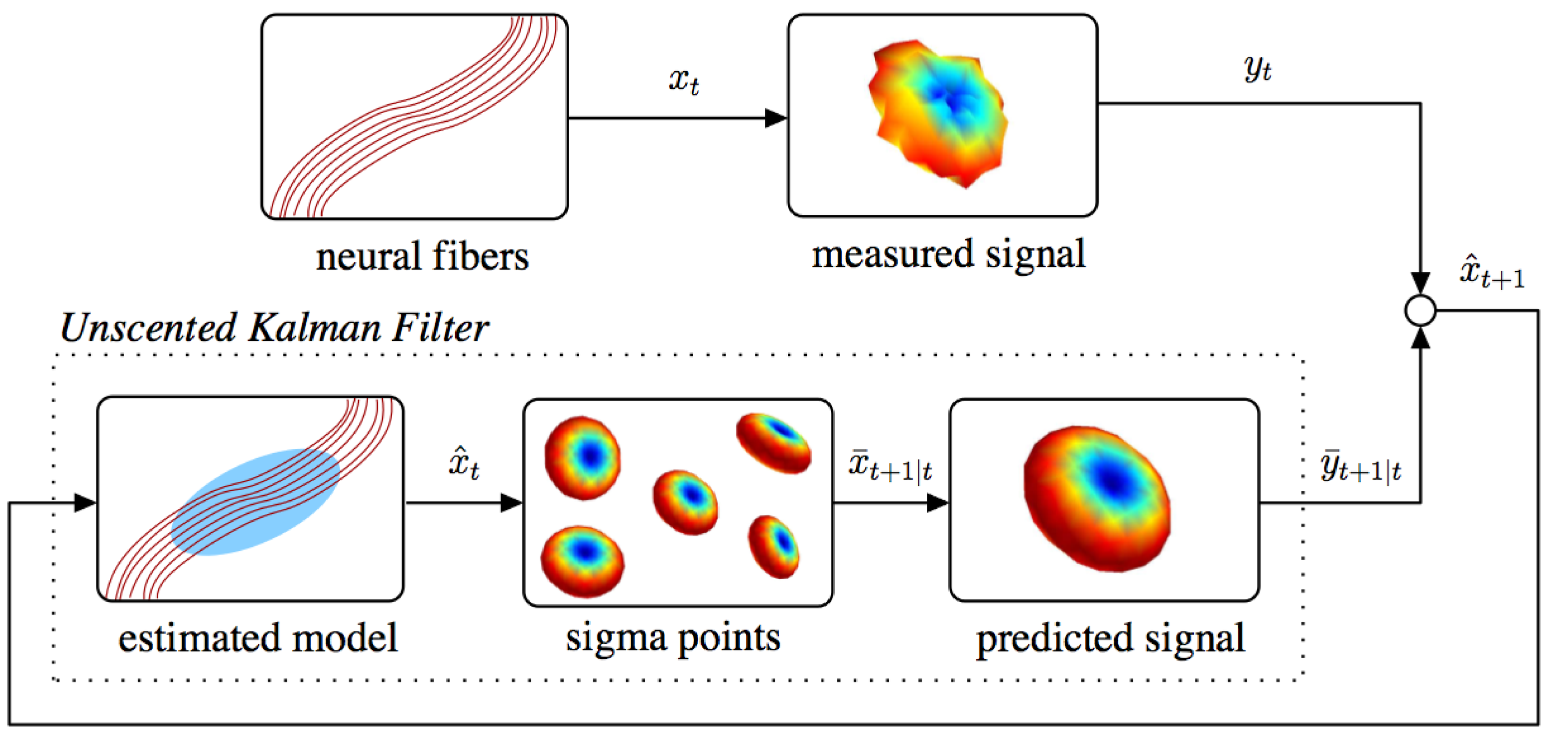
Filter: predict, measure, reconcile, repeat, ...

model state:
$$\boldsymbol{x} = [\boldsymbol{m}_1 \lambda_{11} \lambda_{12} \boldsymbol{m}_2 \lambda_{21} \lambda_{22}]^T \in R^{10}$$

observation/measurement:
$$\boldsymbol{y} = S(\boldsymbol{u}) \quad \text{signal}$$

Unscented Kalman filter (nonlinear signal reconstruction)
use small sample set to estimate mean/cov of nonlinear system

1. spread sigma point set around current mean/cov
2. predict state of each sample (identity)
3. predict observation of each sample (signal reconstruction)
4. measure signal (from dMRI)
5. calculate Kalman gain and update mean/cov estimate



Constraints: solve quadratic penalty weighted by cov

$$\min_{\hat{\boldsymbol{x}}} (\boldsymbol{x} - \hat{\boldsymbol{x}})^T P_t^{-1} (\boldsymbol{x} - \hat{\boldsymbol{x}}) \quad \text{s.t. } A\hat{\boldsymbol{x}} \le \boldsymbol{b}$$

at each step to enforce model constraints

$$\lambda_{11}, \lambda_{21}, \lambda_{12}, \lambda_{22} > 0 \qquad w_1, w_2 > 0 \qquad w_1 + w_2 = 1$$

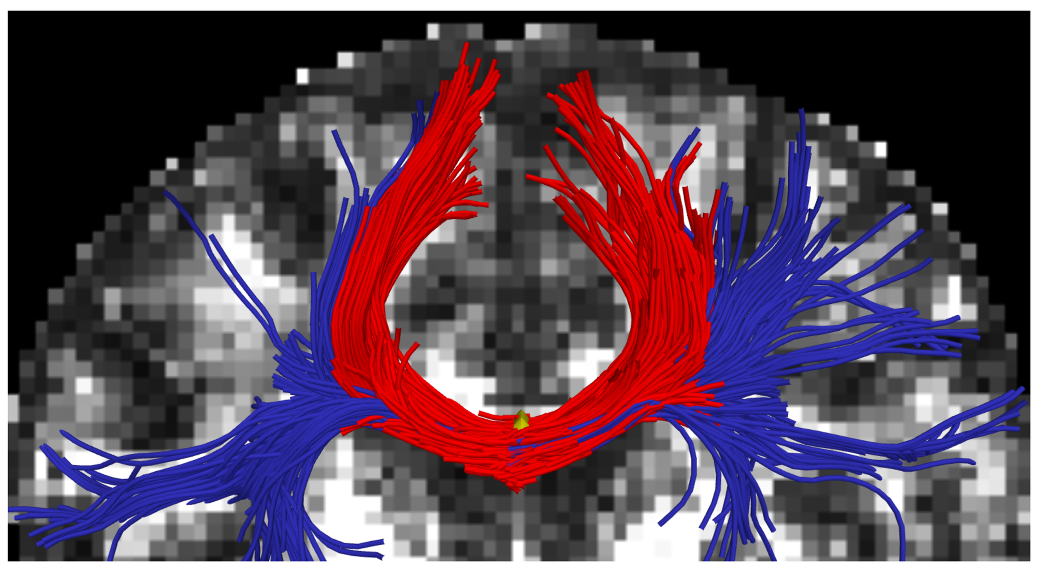
Tractography: at each step, examine the measured signal at that position, use that measurement to update the model parameters within the filter, and propagate forward in the most consistent direction.

Experiments

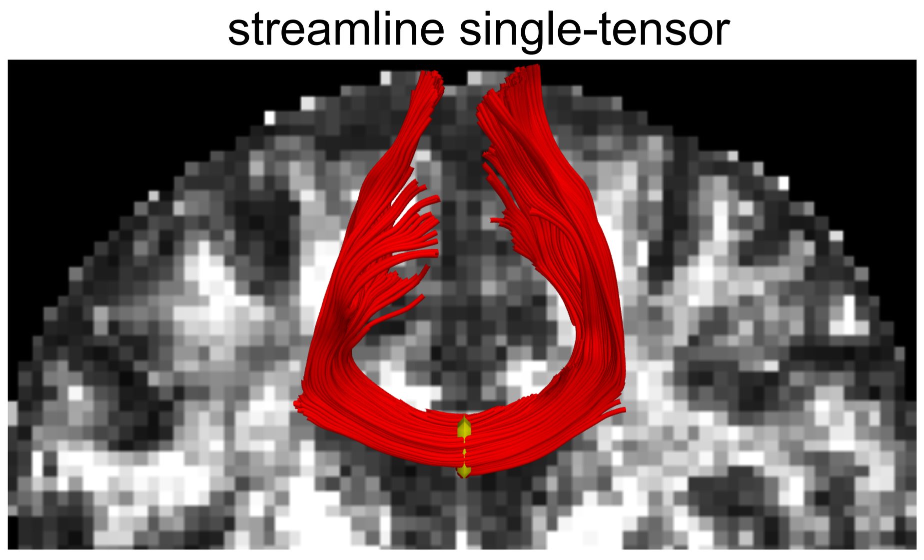
Compare filtered tractography against:
- Single-tensor streamline [Slicer2]
- Sharpened spherical harmonics [Descoteaux 2009]

Experiment: Seed middle of corpus callosum (yellow).
Result: Filtered tractography finds many more lateral paths (blue).

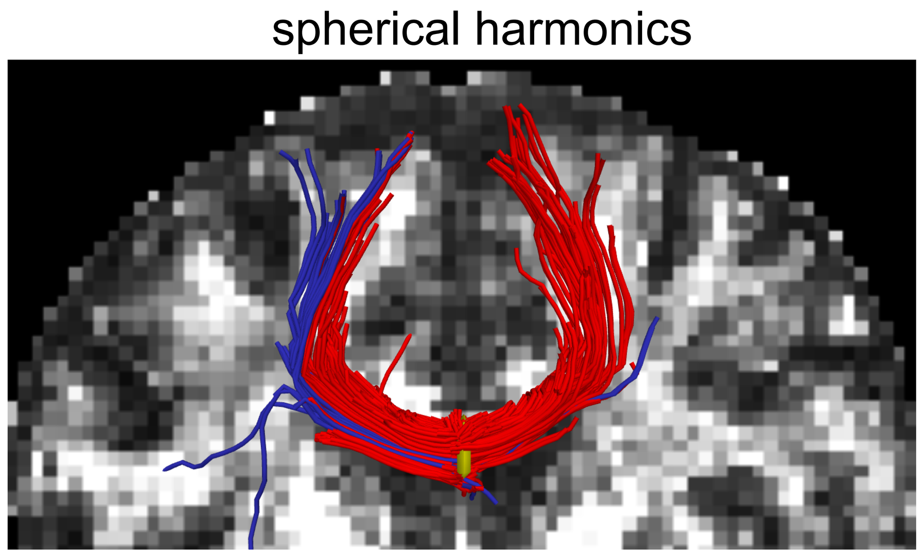
filtered two-tensor



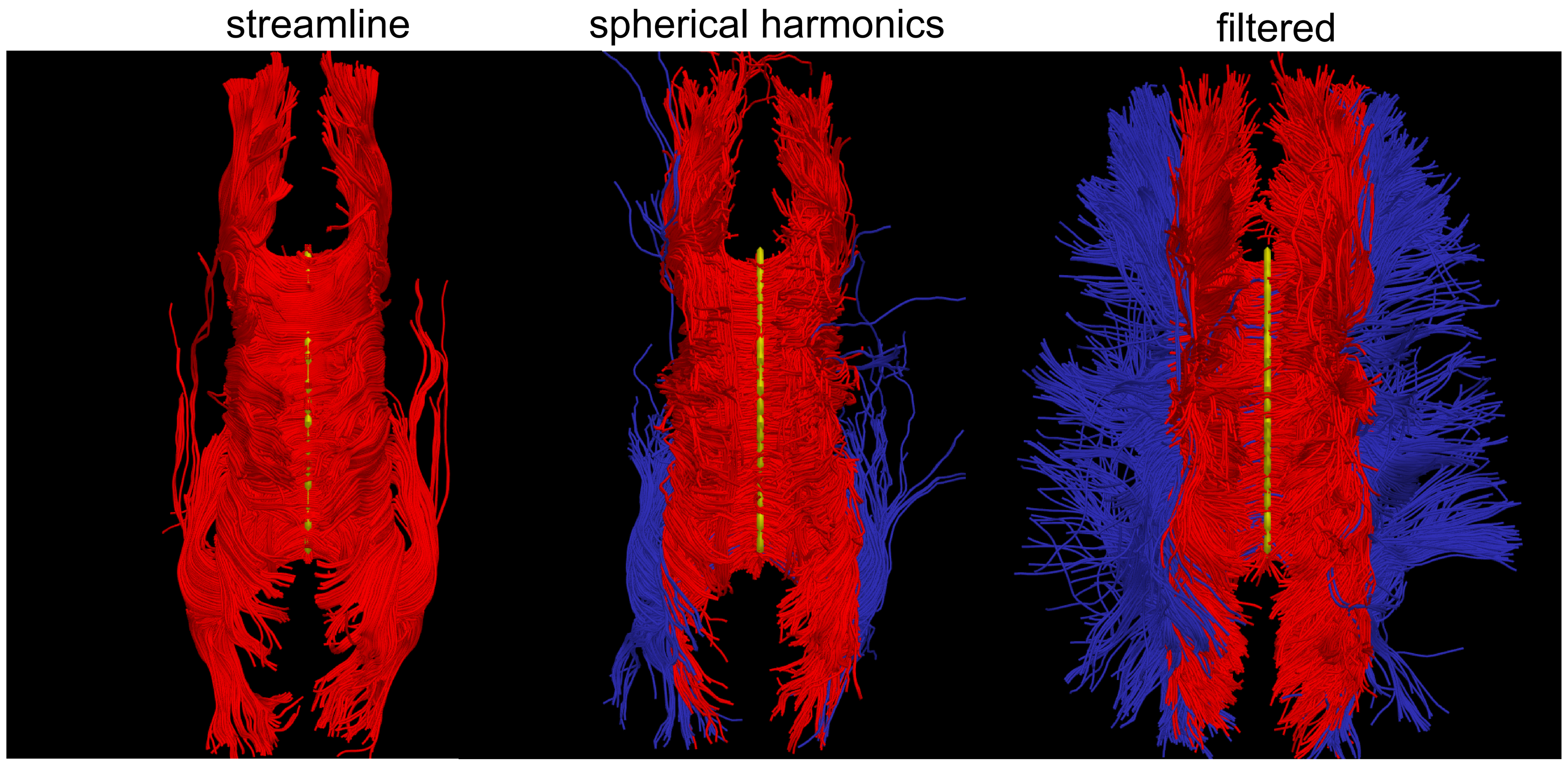
streamline single-tensor



spherical harmonics



streamline spherical harmonics filtered



Experiment: Affinity propagation [Frey2007], Chamfer distance
- Allows non-symmetric distances
- Automatically determines cluster count
- One parameter: relative size of clusters

Result: Two-tensor provides dense gray-matter penetration